WORKING PAPER SERIES

Numéro 02/2023

Portfolio Diversification During Recent Stress and Stress-free Episodes: Insights from Three Alternative Portfolio Methods

Draft version 3
Work in Progress PLEASE DO NOT CITE WITHOUT PERMISSION

Authors
Amine Ben Amar[a,b], Chiheb Féki[b] & Makram Bellalah[b]
[a] *Dept. of Finance, Excelia Business School, 17024, La Rochelle, France.*
[b]*Laboratoire d'Economie, Finance, Management & Innovation - LEFMI [UR 4286], University of Picardie Jules Verne, Amiens, France.*

Abstract

1. Introduction

Diversification is the mixing of financial assets within a portfolio, thereby limiting exposure to any one asset or risk. In the presence of different investment alternatives, investors seek to allocate their wealth in an optimal manner. Markowitz (1952) developed a portfolio theory that would later become the benchmark for all subsequent models in finance. Indeed, he explains that in financial markets, investors allocate their wealth by considering two fundamental characteristics of financial assets: the expected return and the underlying risk. Formally, individuals determine the optimal composition of their portfolios by either setting the level of risk and then choosing the portfolio composition that maximizes return or minimizing the risk for a given level of return. A portfolio made up of different types of assets will, on average, produce higher returns and reduce the specific risk of each individual security or asset.

Asset management shows that international diversification dominates domestic diversification in terms of mean-variance. Indeed, since Grubel (1968), Levy and Sarnat (1970), and Solnik (1974), several research have tried to study the effectiveness of the international diversification strategy (among others: Phylaktis and Ravazzolo 2005; Kellner and Rösch 2019). Such an international strategy aims to improve portfolio performance and reduce volatility. Therefore, this international diversification allows an international investor allocating part of his wealth to some foreign securities to have relatively higher gains than those recorded by an investor investing in purely domestic assets. These gains can be explained by the low correlations and/or the segmentation of certain markets.

The process of financial liberalization has accelerated over the last three decades. This process affects diversification gains. This fact is due to an emergence of financial market integration, linked to an increase in correlation between stock market indices. In the presence of such integration, the effectiveness of diversification is reduced, and investors are therefore attracted by national securities, a phenomenon known as "home bias".[1] However, this financial integration does not concern all financial markets and assets and, as a result, some financial markets can still offer interesting diversification opportunities.

The benefits of international diversification are often attributed to the low correlation between financial markets rather than between individual stocks within a market. However, in recent years, markets have undergone major reforms. These reforms have, for example, lowered the barriers to international investment and, in the process, brought about crucial changes in the financial sector and exacerbated the process of financial integration. Such phenomena have fostered not only similarities in market responses to macro-financial variables, but also connectedness between different financial markets. This similarity has resulted in increased correlations between stock markets and increased systemic risk, affecting asset allocation strategies, and calling into question the effectiveness of diversification.

[1] This irrational behavior can be explained by many factors, in particular, the effect of market segmentation such as investment barriers, transaction costs, taxes, asymmetric information, human capital, investment restrictions and currency risk.

While there is a considerable literature examining domestic and international portfolio strategies, there is still much to be said regarding the role of commodities in diversification strategies.s, especially when it comes to the interaction between commodity and non-commodity assets. A substantial strand of the literature has documented negative correlation between commodity and stock returns (among others: Ankrim and Hensel, 1993; Erb and Harvey, 2006; Erb and Harvey, 2016). While this negative correlation combined with the increased financialization of commodities since the early 2000s has led market participants to view commodities as a new asset class with a fairly high diversification potential, the question of whether commodities offer good diversification potential is still debated in the academic literature. While some studies are optimistic about the benefits of including commodities when structuring asset portfolios (e.g., Gorton and Rouhenhorst, 2006; Belousova and Dorfleitner, 2012; Bhardwaj et al., 2016; Gagnon et al., 2020), others are rather doubtful and cast questions especially after the dramatic increases in the correlation between commodity and equity markets (e.g., Cheung and Miu, 2010; Isleimeyyeh, 2020; Authers and Meyer, 2015; Erb and Harvey, 2016), and others deny the diversification hypothesis (e.g., Belousova and Dorfleitner, 2012; Daskalaki et al., 2014). For instance, Gorton and Rouhenhorst (2006) and Bhardwaj et al. (2016) find commodities provide diversification to a traditional portfolio of stocks and bonds, particularly during periods of high volatility. Belousova and Dorfleitner (2012) results' suggest that the diversification contribution varies greatly amongst the different categories of commodities. Indeed, while some including some commodities (e.g. industrial metals, agricultural and livestock) contribute to the reduction of the level of risk, including others (e.g., energy and precious metals) contribute not only to the reduction of the level of risk, but also to the improvement of the portfolio's return. More recently, Gagnon et al. (2020) use spanning test to investigate the diversification potential of commodities in Canada over the period 1993-2019. Their results suggest that including commodities significantly improve portfolio performance during the post-financialization period. On the other hand, Cheung and Miu (2010) demonstrate that the diversification benefit of commodities is a far more complex phenomenon than often understood in the finance literature. Authers and Meyer (2015) argue that exposure to commodity futures is beneficial for large investors only. As for Erb and Harvey (2016), they conclude their article by stating that "*the appeal of commodities for investors is unlikely to reside in easy misperceptions that commodities are an inflation hedge, a portfolio diversifier, or a source of a "risk-transfer" risk premium*".

Investigating diversification strategies combining different financial asset classes may significative financial implications for investors: Not only does it guide them in rebalancing their portfolios and developing appropriate investment strategies during stress and stress-free periods, but it also reinforces the productive use of funds within the financial system. Moreover, exploring and understanding portfolio strategies by combining pure financial assets and commodities is very important for all market participants as commodities differ from other financial assets in many ways. First, commodity prices are generally more volatile than other financial assets (Dwyer et al., 2011). Second, commodity futures cover several maturities. This makes them appropriate for short- and long-term investments. Thus, this characteristic allows investigating diversification strategies by using short-term and/or long-term commodity futures. Third, since the beginning of the 2000s, commodity futures markets have become ever more integrated with stock markets, due to the increasing use of these assets as diversification and hedging tools (Gorton and Rouwenhorst, 2006; Babalos and Stavroyiannis, 2015).

The objective of this paper is to examine the structure as well as the performance of different investment strategies using different portfolio methods. More specifically, we first use a time-varying parameter vector autoregressive model (TVP-VAR model) *à la* Antonakakis et al. (2020) to estimate time-varying variance-covariance matrices, and the Diebold and Yilmaz (2012) Generalized Forecast Error Variance Decomposition (GFEVD) framework to examine the connectedness structure among the considered stock and commodity markets. Then, we construct different portfolios using three diversification strategies – the traditional minimum-variance portfolio strategy (Markowitz, 1952), the minimum-correlation portfolio strategy (Christoffersen, et al., 2014), and the minimum-connectedness portfolio strategy (Broadstock et al., 2022) – and compare them.

To that end, we assume an investor interested in investing in (**i**) the global equity market, but also in (**ii**) regional equity markets (North America, Latin America, Europe, Asia-Pacific, GCC) and (**iii**) short- and long-term commodity futures markets (Oil, Natural Gas, Soybeans, Soybean Oil, Sugar, Wheat, Copper, Gold)[2] over the period from January 1st, 2019 to October 14th, 2022. Specifically, we assume that the investor has a choice among five different investment strategies, and we examine the structure and performance of these strategies during stress and stress-free periods.

We have uncovered several results which can be summarized as follows. First, the overall connectedness network analysis reveals a relatively high level of integration among global and regional stock markets, and between each of the short-term futures contracts and its long-term counterpart, supporting the thesis stating that stock and commodity markets are potentially weakly integrated (Belke and Dubova, 2018), however the regional stock markets appear to be relatively much more integrated than commodity futures markets. Moreover, the overall connectedness analysis show that the natural gas market seems to be the most isolated market, i.e., it weakly affects (and is weakly affected by) other markets. Second, agricultural commodity markets appear to be broadly insensitive to shocks on non-agricultural commodity markets, which support the thesis stating that commodity markets are potentially segmented (Hammoudeh et al. 2010; Gardebroek et al. 2016). Third, the results suggest a relatively high connectedness among pure financial markets on average and across the entire sample period, and a relatively low connectedness among regional stock markets and long-term commodity futures markets. Fourth, the connectedness analysis show that the GCC stock market is largely disconnected from other regional financial markets, as it does not influence nor is it influenced by other regional stock markets to any great extent. Fifth, the results suggest that there is no dominant strategy during the pre-COVID stress-free period and through mid-2019, i.e., that the performance of the different investment strategies were largely equivalent in terms of cumulative returns. Furthermore, from mid-2019 to mid-2021, and going through the COVID-19 period, the portfolio strategy composed of regional indices and short-term

[2] Very few papers in the literature highlight the importance of putting attention on short- as well as long-term commodity futures (see among others: Buyuksahin and Robe, 2014; Isleimeyyeh, 2020; Ben Amar et al. 2022).

commodity futures contracts, under the Markowitz (1952) minimum-variance approach, outperforms the other investment strategies. However, we further find that the same portfolio strategy, but under the Broadstock et al. (2022) minimum-connectedness approach, outperforms all other investment strategies from mid-2021and during the ongoing Russian-Ukrainian war period.

The main contributions of this study to the literature are twofold. First, our paper fills the gap in the literature by examining diversification strategies when considering both stock markets and commodities over different maturities, ranging from the short- to the long-term, as this may hide useful information in terms of fund allocation and risk management. Second, the period under study includes recent episodes of high market uncertainty, including the COVID-19 period and the ongoing Russian-Ukrainian conflict, as well as periods of low market uncertainty. This allows us to track the dynamics of portfolio weights structure during stress as well as stress-free episodes. Our findings are of practical importance for market participants. Indeed, in a context of increasing financial integration, a better understanding of potential diversification opportunities, as well as their implications in terms of risk and return, in both crisis and calm periods, is crucial for investors.

The remainder of the article is structured as follows. Section 2 explains the methodology and describes the dataset used as well as the different portfolio strategies. Section 4 analyzes the results. Section 5 concludes the study and brings practical implications.

2. Empirical Strategy

2.1. Methodology

Our empirical strategy consists of two complementary steps. First, we use a time-varying parameter vector autoregressive model (TVP-VAR model) *à la* Antonakakis et al. (2020) to estimate time-varying variance-covariance matrices, and the Diebold and Yilmaz (2012) Generalized Forecast Error Variance Decomposition (GFEVD) framework to examine the connectedness structure among the considered stock and commodity markets. Second, we construct different portfolios using three diversification strategies: the traditional minimum-variance portfolio strategy (Markowitz, 1952 & 1959), the minimum-correlation portfolio strategy (Christoffersen, et al., 2014), and the minimum-connectedness portfolio strategy (Broadstock et al., 2022).

2.1.i. TVP-VAR-based connectedness

The vector autoregression (VAR) approach proposed by Sims (1980) provides an appropriate framework for connectedness and spillovers analysis (Diebold and Yilmaz, 2009 & 2012). This approach has become a standard econometric tool in the economic and financial literature. However, constant-parameters VAR models may provide inaccurate and incomplete information when the structure of the relationship among the variables studied is not stable over time (Canova and Pérez Forero, 2015; Antonakakis et al., 2020).

By the end of the 1990s, the methodology of VAR models had been extended to include time-varying components. In 2001, Cogley and Sargent introduced a time-varying VAR model to investigate the persistence of inflation in the United States over the post-war period (Cogley and Sargent, 2001). Unlike standard constant-parameters VAR models, the TVP-VAR model does not require the initial sample to be decomposed into subsamples to confirm or deny the change in model structure. Indeed, rather than decomposing the whole sample into subsamples, the fact that the coefficients vary over time allows to date shifts and transitions accurately. Thus, TVP-VAR models prevent information loss due to the decomposition of the sample, as well as the risk of having results that depend on the largely arbitrary choice of sub-samples. Due to the neglect of possible time-varying variance of structural shocks, Sims (2001) claims that the findings of Cogley and Sargent (2001) are probably exaggerated.

To address this issue, Primiceri (2005) suggests a VAR model that allows all parameters to vary over time. In the Primiceri (2005) model's, time variation concerns the parameters of the model, as well as the variance-covariance matrix of errors (stochastic volatility). Allowing the variance-covariance matrix to vary over time in TVP-VAR models improves the performance of estimation by tracking potential changes in the structure of the system in a more flexible and robust manner (Nakajima, 2011). Thus, the model proposed by Primiceri (2005) captures possible nonlinearities in the interactions among the variables in the model, as well as the heteroscedasticity of the errors that may be due to changes in the size of exogenous shocks or their impacts on the variables (Koop et al, 2009; Koop and Korobilis, 2010; D'Agostino et al. 2013). Thus, unlike constant-parameters VAR models, which obtain the impulse responses under the assumption that the coefficients do not vary over the horizon of the impulse responses, TVP-VAR models introduce a new dimension that allows to both study the interaction between variables, as well as the impulse responses at different points in time, and to better understand the transmission of shocks.

Since Primiceri's (2005) TVP-VAR model allows both shifts in the structure of the system and the volatility of shocks to be considered, it has been used in several empirical studies on the dynamics of the structure of the economy (among others: Baumeister et al. 2008; Nakajima, 2011; Bijsterbosch and Falagiarda, 2014; Carriero et al. 2015; Canova and Pérez Forero, 2015; He and Lin, 2018), as well as on the connectedness among financial markets (Antonakakis et al., 2020) and portfolio management (Broadstock et al., 2022). Thus, we use in a first step the Antonakakis et al. (2020) methodology of TVP-VAR-based connectedness to analyze the connectedness among commodity and equity markets.

By using a multivariate Kalman Filter TVP-VAR model *à la* Koop and Korobilis (2014), Antonakakis et al. (2020) build on and extend the connectedness of framework originally proposed by Diebold and Yilmaz (2012) by letting the variance-covariance matrix vary over time. Their approach has three major advantages relative to that of Diebold and Yilmaz (2012): it avoids **(i)** making a mostly arbitrary choice of the size of the rolling window, as well as **(ii)** the loss of valuable observations, and **(iii)** overcomes the burden of the results being sensitive to outliers.

A TVP-VAR model of order 1 is given by:[3]

$$y_t = \alpha_t y_{t-1} + \varepsilon_t \qquad \varepsilon_t | Z_{t-1} \sim N(0, \Sigma_t) \qquad (1)$$
$$vec(\alpha_t) = vec(\alpha_{t-1}) + \omega_t \qquad \omega_t | Z_{t-1} \sim N(0, \Omega_t) \qquad (2)$$

[3] In this paper we provide a summary of the key econometric structure of the TVP-VAR, and refer interested readers to the article by Antonakakis et al. (2020) and Broadstock et al. (2022) for further technical details. Without loss of generality and for the sake of simplicity, we present here a TVP-VAR model of order 1. However, the Bayesian information criterion (BIC) also indicates that this is the appropriate lag order for our different investment strategies (see Section 4).

where Z_{t-1} represents all information available up to $t-1$, $y_t = (y_{1t}, \dots, y_{kt})'$ is the $k \times 1$ dimensional vector of endogenous variables, $\alpha_t = (\alpha_{1t}, \dots, \alpha_{kt})'$ the the $k \times k$ dimensional matrices of coefficients. $vec(\alpha_t)$, which is the vectorization of α_t, is an $k^2 \times 1$ dimensional vector. The disturbance ε_t is a $k \times 1$ dimensional vector of orthogonal structural shocks, and ω_t is an $k^2 \times 1$ dimensional vector. The time-varying variance-covariance matrices Σ_t and Ω_t are $k \times k$ and $k^2 \times k^2$ dimensional matrices, respectively. It is broadly admitted that time-series contain time-conditional heteroskedasticity and allowing variances to vary over time not only helps to manage this, but also improves estimation performance by capturing potential changes in the structure of the system. Similar to Gabauer (2021) and Broadstock et al. (2022), a VAR(1) estimated on the first 200 observations is used to derive prior means and variances needed to initialize the Kalman filter.

By using the the time-varying parameters and variance-covariance matrices in accordance with Koop and Korobilis (2014), as well as the generalized impulse response functions (GIRFs) and the generalized forecast error variance decompositions (GFEVDs) in accordance with Koop et al. (1996) and Pesaran and Shin (1998), Antonakakis et al. (2020) build on and extend the generalized connectedness measures of Diebold and Yilmaz (2012). To compute the GIRFs and the GFEVDs, we need to derive the moving-average representation of the stationary time-varying vector autoregressive process. According to the Wold representation theorem (Wold, 1954), the moving-average representation of the TVP-VAR model is given by $y_t = \sum_{j=0}^{\infty} \Lambda_{jt}\, \varepsilon_{t-j}$, where Λ_{jt} is a $k \times k$ dimensional matrix.

The GIRFs depict the responses of all variables within the system following a shock to variable j. They are obtained from the moving-average representation of the TVP-VAR model, as the difference between the conditional and unconditional forecast, where the conditioning information set is the shock to the j^{th} variable (koop et al., 1996). Let $H = \{1, 2, 3, \dots\}$ be the forecast horizon, $i = \{1, 2, \dots, k\}$ count the variables included in the system, δ_j the shock to the j^{th} variable in the system, where $j = \{1, 2, \dots, k\}$, and Z_{t-1} represents all information available at time $t-1$. The GIRFs is then defined by

$$GIRF_t(i, t+H, \delta_{j,t}, Z_{t-1}) = E(y_{i,t+H} | u_{j,t} = \delta_{j,t}, Z_{t-1}) - E(y_{i,t+H} | Z_{t-1}) \tag{3}$$

Based on Peseran and Shin (1998), the orthogonalized and generalized impulse response functions to a shock on the j^{th} equation ($\psi_{j,t}(H)$) is given by

$$\psi_{j,t}(H) = \Sigma_{jj,t}^{-\frac{1}{2}} \Lambda_{H,t} \Sigma_t e_j \tag{4}$$

where Σ_{jj} is the (j, j) element of Σ_t, Λ_H is the H^{th} coefficient-matrix from the moving-average representation of the TVP-VAR model, and e_j is a $k \times 1$ dimension selection vector having unity in the j^{th} position, and zeros otherwise. Thus, the GFEVD ($\tilde{\lambda}_{ij,t}^{G}(H)$), which represents the directional pairwise connectedness from variable j to variable i at horizon H, is given by

$$\tilde{\lambda}_{ij,t}^{G}(H) = \frac{\sum_{t=1}^{H-1} \psi_{ij,t}^2}{\sum_{j=1}^{k} \sum_{t=1}^{H-1} \psi_{ij,t}^2} \tag{5}$$

By using the GFEVD, which measures how much of the variance forecast error of variable i, at horizon H, is due to shocks on variable j, the total connectedness index, CI_t, which summarizes the overall interdependence among variables in the system, is defined as follows

$$\mathrm{CI}_t^H = \frac{\sum_{\substack{i,j=1\\ i\neq j}}^{k} \tilde{\lambda}_{ij,t}^{G}(H)}{k-1} \cdot 100, \qquad 0 \leq \mathrm{CI}_t^H \leq 1 \tag{6}$$

The total connectedness index can be decomposed into directional connectedness indices. Specifically, the directional connectedness to variable i received from all remaining variables, $\mathrm{CI}_{i\leftarrow\bullet,t}^{H}$, is defined by

$$\mathrm{CI}_{i\leftarrow\bullet,t}^{H} = \frac{\sum_{\substack{j=1\\ j\neq i}}^{k} \tilde{\lambda}_{ij}^{G}(H)}{k} \cdot 100 \tag{7}$$

and, similarly, the directional connectedness transmitted from variable i to all remaining variables, $\mathrm{CI}_{i\rightarrow\bullet,t}^{H}$, is given by

$$\mathrm{CI}_{i\rightarrow\bullet,t}^{H} = \frac{\sum_{\substack{j=1\\ j\neq i}}^{k} \tilde{\lambda}_{ji}^{G}(H)}{k} \cdot 100 \tag{8}$$

The difference between $\mathrm{CI}_{i\rightarrow\bullet,t}^{H}$ and $\mathrm{CI}_{i\leftarrow\bullet,t}^{H}$ is called net connectedness. Thus, the net connectedness, $\mathrm{NCI}_{i,t}^{H}$, can be obtained from equations (7) and (8) as follows

$$\mathrm{NCI}_{i,t}^{H} = \mathrm{CI}_{i\rightarrow\bullet}^{H} - \mathrm{CI}_{i\leftarrow\bullet}^{H} \tag{9}$$

Which simply indicates whether a market i is a net receiver or a net transmitter of volatility shocks. If $\mathrm{NCI}_{i,t}^{H} > 0$, then market i influences the other markets more than it is being influenced by them. In contrast, if $\mathrm{NCI}_{i,t}^{H} > 0$, then market i is influenced by the other markets more than it influences them.

According to Gabauer (2021), the pairwise connectedness index, $\mathrm{PCI}_{ij,t}^{H}$, which measures the interconnectedness between variables i and j, is given by

$$\mathrm{PCI}_{ij,t}^{H} = 2 \cdot \left(\frac{\tilde{\lambda}_{ij,t}^{G}(H) + \tilde{\lambda}_{ji,t}^{G}(H)}{\tilde{\lambda}_{ii,t}^{G}(H) + \tilde{\lambda}_{ij,t}^{G}(H) + \tilde{\lambda}_{ji,t}^{G}(H) + \tilde{\lambda}_{jj,t}^{G}(H)} \right), \qquad 0 \leq \mathrm{PCI}_{ij,t}^{H} \leq 1 \tag{10}$$

This measure illustrates the degree of bilateral connection among variables i and j, and it ranges between 0 and 1.

2.1.ii. Portfolios structuring approaches

We consider in this subsection several approaches to portfolio construction, including the traditional Markowitz (1952 & 1959) approach, as well as more recent correlation (Christoffersen et al. 2014) and connectedness (Broadstock et al. 2022) oriented approaches. In what follows, we provide brief summaries of the approaches we use.

- *Minimum-variance portfolio*

The minimum-variance portfolio approach introduced by Markowitz (1952 & 1959), which consists in structuring the portfolio that provide the lowest possible variance among all possible portfolios of risky assets, is one of the most used approaches in portfolio management. According to this approach, the respective weights of the different assets in the portfolio, $\upsilon_t^{MV} = \left(\upsilon_{1,t}^{MV}, \dots, \upsilon_{k,t}^{MV}\right)'$, are given by

$$\upsilon_t^{MV} = \frac{\Sigma_t^{-1}\mathbb{I}}{\mathbb{I}^T\Sigma_t^{-1}\mathbb{I}} \tag{11}$$

where υ_t^{MV} is a $k \times 1$ dimensional vector of portfolio weights, $\mathbb{I}$ is a $k \times 1$ dimensional vector whose entries are ones, and Σ_t is the $k \times k$ dimensional conditional variance-covariance matrix in period t.

The return generated by this portfolio at each time t, μ_t^{MV}, is obtained by

$$\mu_t^{MV} = \frac{\mu_t'\Sigma_t^{-1}\mathbb{I}}{\mathbb{I}^T\Sigma_t^{-1}\mathbb{I}} \tag{12}$$

where μ_t' is the $k \times 1$ dimensional vector of the returns of the underlying assets at time t.

The return variance, σ_t^{MV}, of this global minimum variance portfolio is given by

$$\sigma_t^{MV} = \frac{1}{\mathbb{I}^T\Sigma_t^{-1}\mathbb{I}} \tag{13}$$

- *Minimum-correlation portfolio*

Christoffersen et al. (2014) propose using the conditional correlation matrix instead of the variance-covariance matrix to compute the weights vector. According to this approach, the respective weights of the different assets in the portfolio, $\upsilon_t^{MC} = \left(\upsilon_{1,t}^{MC}, \dots, \upsilon_{k,t}^{MC}\right)'$, are given by

$$\upsilon_t^{MC} = \frac{\rho_t^{-1}\mathbb{I}}{\mathbb{I}^T\rho_t^{-1}\mathbb{I}}, \qquad \text{with} \qquad \rho_t = \mathrm{diag}(\Sigma_t)^{-\frac{1}{2}}\ \Sigma_t\ \mathrm{diag}(\Sigma_t)^{-\frac{1}{2}} \tag{14}$$

where ρ_t is the $k \times k$ dimensional conditional correlation matrix in period t.

The return generated by this portfolio at each time t, μ_t^{MC}, is obtained by

$$\mu_t^{MC} = \frac{\mu_t'\rho_t^{-1}\mathbb{I}}{\mathbb{I}^T\rho_t^{-1}\mathbb{I}} \tag{15}$$

where μ_t' is the $k \times 1$ dimensional vector of the returns of the underlying assets at time t.

The return variance, σ_t^{MC}, of this global minimum correlation portfolio is given by

$$\sigma_t^{MV} = \left(\upsilon_t^{MC}\right)^T\Sigma_t\upsilon_t^{MC} \tag{16}$$

- *Minimum-connectedness portfolio*

More recently, and instead of using the conditional variance-covariance matrix (Markowitz, 1952 & 1959) or the conditional correlation matrix (Christoffersen et al., 2014), Broadstock et al. (2022) propose using the pairwise connectedness indices to derive the vector of weights. Minimizing the connectedness among the variables within the system helps structure a portfolio that is more resilient to systemic risk. Thus, the most isolated underlying assets (i.e. those that are less influenced by others, and that influence others less), will have a greater weight in the portfolio. According to this approach, the respective weights of the different assets in the portfolio, $\upsilon_t^{CON} = \left(\upsilon_{1,t}^{CON}, \dots, \upsilon_{k,t}^{CON}\right)'$, are given by

$$\upsilon_t^{CON} = \frac{PCI_t^{-1}\mathbb{I}}{\mathbb{I}^T PCI_t^{-1}\mathbb{I}} \tag{17}$$

where υ_t^{CON} is a $k \times 1$ dimensional vector of weights, $\mathbb{I}$ is a column vector of ones, and PCI_t is the $k \times k$ dimensional pairwise connectedness matrix in period t.

The return generated by this portfolio at each time t, μ_t^{CON}, is obtained by

$$\mu_t^{CON} = \frac{\mu_t' PCI_t^{-1}\mathbb{I}}{\mathbb{I}^T PCI_t^{-1}\mathbb{I}} \tag{18}$$

where μ_t' is the $k \times 1$ dimensional vector of the returns of the underlying assets at time t.

The return variance, σ_t^{CON}, of this global minimum connectedness portfolio is given by

$$\sigma_t^{CON} = \left(\upsilon_t^{CON}\right)^T \Sigma_t \upsilon_t^{CON} \tag{19}$$

2.2. Data

Our underlying datasets are daily observations of the MSCI ACWI & Frontier Markets index [**WRD**], which is a representative global stock market index, five regional stock markets indices (MSCI Europe Index [**EUR**]; MSCI North America Index [**NAM**]; MSCI Asia Pacific Index [**APC**]; MSCI Emerging Markets Latin America Index [**LAM**]; MSCI GCC Countries Index [**GCC**])[4], and a set of commodity futures prices (WTI crude oil [**WTI**], natural gas [**NGAS**], soybeans [**SOY**], soybean oil [**SOL**], sugar [**SGR**], wheat [**WHT**], copper [**CPR**], and gold [**GOLD**]) for the shortest (i.e. the front month) and longest possible maturities.[5] Table 1 depicts the composition of the global and regional stock market indices. The data are collected from Bloomberg, and cover the period running from January 1st, 2019 to October 14th, 2022, thereby providing a sample of 989 trading days. The period studied is informative in terms of market development because it contains both stress and stress-free periods, and more precisely the COVID-19 crisis and the Russia-Ukraine war, in which systemic shocks can be transmitted between the financial markets considered with different magnitudes. All series are expressed in US dollars. The daily return for each stock market index and commodity futures i is computed as the logarithmic return $R_t^i = \ln\left(P_t^i\right) - \ln\left(P_{t-1}^i\right)$, where P_t^i is the daily close price of stock market or commodity futures i on day t. Some descriptive statistics on stock markets and commodity futures returns are presented in Table 2.

[4] For further details on the composition of the global as well as regional stock markets indices, we would refer reader to *msci.com*.

[5] For each of the long-run futures prices, the choice of the maturity was subject to the availability of comprehensive and complete data sets starting January 1st, 2019. For instance, we have selected the WTI crude oil futures with maturity 70 months, the NYMEX Natural Gas futures with maturity 70 months, the CBOT wheat futures with maturity 29 months, the CBOT soybeans futures with maturity 29 months, the CBOT soybean oil futures with maturity 31 months, the ICE sugar #11 futures with maturity 22 months, the COMEX copper futures with maturity 10 months, and the COMEX gold futures with maturity 24 months.

Table 1 MSCI Global and Regional Markets Classification

Indices			Included countries*
MSCI ACWI & Frontier Markets Index**	MSCI ACWI Index	MSCI World Index	United States (69.53%), Japan (6.04%), United Kingdom (4.24%), Canada (3.49%), France (3.06%), Other (13.63%)
		MSCI Emerging Markets Index	China (26.89%), India (16.21%), Taiwan (13.52%), South Korea (11.91%), Brazil (6.45%), Other (25.03%)
	MSCI Frontier Markets Index		Vietnam (30.3%), Morocco (9.61%), Iceland (8.86%), Kazakhstan (8.38%), Romania (6.62%), Other (25.73%)
MSCI Europe Index			United Kingdom (24.11%), France (18.11%), Switzerland (16.19%), Germany (12.24%), Netherlands (6.36%), Other (22.99%)
MSCI North America Index			United States (95.24%), Canada (4.76%)
MSCI Asia Pacific Index			Japan (33.05%), China (17.36%), Australia (11.93%), India (10.46%), Taiwan (8.73%), Other (18.46%)
MSCI Emerging Markets Latin America Index			Brazil (63.54%), Mexico (25.93%), Chile (6.11%), Peru (2.83%), Colombia (1.59%)
MSCI GCC Countries Index			Saudi Arabia (61.88%), United Arab Emirates (14.63%), Qatar (12.47%), Kuwait (10.39%), Oman (0.37%), Other (0.25%)

Notes: *Included countries and weights as of November 8th, 2022; **The MSCI ACWI & Frontier Markets Index captures the performance of large and mid-cap stocks across 23 developed, 24 emerging and 28 frontier markets countries.

Source: www.msci.com/market-classification

Table 2 Descriptive Statistics

	Market	Mean	Median	Max	Min	Std. Dev.	Skew.	Kur.	J-B	ADF
Stock markets	**WRD**	0.0002	0.0007	0.08	-0.10	0.0115	-1.27	18.73	10466***	-8.9***
	NAM	0.0004	0.0007	0.09	-0.13	0.0142	-1.00	17.54	8883***	-9.0***
	EUR	0.0000	0.0009	0.08	-0.13	0.0132	-1.12	16.43	7635***	-30.4***
	APC	-0.0001	0.0004	0.05	-0.06	0.0099	-0.28	6.61	549***	-27.7***
	LAM	-0.0002	0.0005	0.11	-0.16	0.0192	-1.40	16.79	8157***	-10.3***
	GCC	0.0003	0.0001	0.05	-0.17	0.0105	-4.95	75.66	221610.60***	-10.8***
	Market	Mean	Median	Max	Min	Std. Dev.	Skew.	Kur.	J-B	ADF
Short-run commodity futures	**WTI1**	0.0006	0.0019	0.32	-0.35	0.0390	-0.92	27.38	24623***	-31.0***
	GAS1	0.0008	0.0000	0.38	-0.30	0.0415	0.37	14.50	5474***	-34.8***
	SOY1	0.0005	0.0008	0.06	-0.11	0.0139	-1.03	11.36	3051***	-30.6***
	SOL1	0.0009	0.0000	0.07	-0.09	0.0180	-0.52	5.30	262***	-28.9***
	SGR1	0.0005	0.0000	0.08	-0.08	0.0168	0.08	4.62	108***	-31.9***
	WHT1	0.0005	0.0000	0.20	-0.11	0.0217	0.66	11.82	3278***	-31.0***
	CPR1	0.0003	0.0000	0.05	-0.07	0.0143	-0.33	4.59	122***	-32.1***
	GLD1	0.0003	0.0004	0.06	-0.05	0.0101	-0.30	7.87	992***	-31.1***
	Market	Mean	Median	Max	Min	Std. Dev.	Skew.	Kur.	J-B	ADF
Long-run commodity futures	**WTI70**	0.0001	0.0000	0.05	-0.07	0.0125	-0.40	5.96	386***	-34.1***
	GAS70	0.0004	0.0000	0.08	-0.12	0.0127	-1.81	25.27	20976***	-31.0***
	SOY17	0.0003	0.0000	0.09	-0.06	0.0089	0.58	22.31	15421***	-31.4***
	SOL20	0.0006	0.0000	0.10	-0.09	0.0134	-0.25	14.57	5522***	-24.1***
	SGR8	0.0002	0.0000	0.04	-0.05	0.0093	-0.29	5.74	323***	-32.0***
	WHT12	0.0003	0.0000	0.09	-0.27	0.0147	-6.20	125.89	628658***	-34.1***
	CPR10	0.0002	0.0001	0.05	-0.07	0.0131	-0.42	5.00	193***	-31.9***
	GLD12	0.0003	0.0000	0.07	-0.06	0.0085	-0.87	16.62	7764***	-32.3***

Note: **WRD**, **NAM**, **EUR**, **LAM**, **APC**, and **GCC** stand for MSCI ACWI Index, MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. **WTI1** stands for front NYMEX WTI crude oil futures contract. **GAS1** stands for front NYMEX Natural Gas futures contract. **WHT1** stands for front CBOT wheat futures contract. **SOY1** stands for front CBOT soybeans futures contract. **SOL1** stands for front CBOT soybean futures contract. **SGR1** stands for front ICE sugar #11 futures contract. **CPR1** stands for front COMEX copper futures contract. **GLD1** stands for front COMEX gold futures contract. **WTI70** stands for NYMEX WTI crude oil futures with maturity 70 months. **GAS70** stands for NYMEX Natural Gas futures with maturity 70 months. **WHT12** stands for CBOT wheat futures with maturity 29 months. **SOY17** stands for CBOT soybeans futures with maturity 29 months. **SOL20** stands for CBOT soybean futures with maturity 31 months. **SGR8** stands for ICE sugar #11 futures with maturity 22 months. **CPR10** stands for COMEX copper futures with maturity 10 months. **GLD12** stands for COMEX gold futures with maturity 24 months. *** (**) denotes level of significance at the 1 per cent (5 per cent) level.

Table 2 depicts descriptive statistics for the stock markets and commodity futures returns. The highest level of return is observed in the **SOL1** market, while the **LAM**, **APC** and **EUR** markets report the lowest returns. The **GAS1** market has the highest volatility with a value of 0.0415, followed by the **WTI1** market with a value of 0.039. The **GLD12** market has the lowest volatility with a value of 0.0085. Since the mean is lower than the median for all the stock markets returns, except for the **GCC** stock market, the mean is being pulled downward by extreme values (i.e., there are more extreme lower values than higher values). For the **GCC** return, we notice that the mean is higher than the median which implies that the mean is pulled upward by extreme values (i.e., there are more extreme higher values than lower values). For long-run commodity futures, we notice that there are more extreme higher values than lower values (as the mean is higher than the median for all the long-run commodity futures returns). Regarding short-run commodity futures, some are pulled upward by extreme values (**GAS1**, **SOL1**, **SGR1**, **WHT1**, **CPR1**) and others are pulled downward (**WTI1**, **SOY1**, **GLD1**)

Standard deviation of market returns ranges from 0.0085 for **GLD1** to 0.039 and 0.0415 for **WTI1** and **GAS1**, respectively. The distributions of the variables considered are leptokurtic (i.e., they have heavier tails and a sharper peak than the normal distribution) and asymmetric, with positive values of skewness for **GAS1**, **SGR1**, **WHT1** and **SOY17** (i.e., the right tail of their distributions is larger than the left tail), and negative values for the rest of the variables (i.e., the left tail of their distributions is larger than the right tail), rejecting the normality property for the return series, which is confirmed by the Jarque-Bera statistic (at the 1% significance level). The ADF unit root test suggests that all the return series are stationary at the 1% significance level.

2.3. Diversifications Strategies

We assume an investor interested in investing in (**i**) the global equity market, but also in (**ii**) regional equity markets (North America, Latin America, Europe, Asia-Pacific, GCC) and (**iii**) commodity futures markets (Oil, Natural Gas, Soybeans, Soybean Oil, Sugar, Wheat, Copper, Gold). Regarding stock market indices, we assume that investors can acquire the indices via trackers or other investment vehicles. Specifically, we assume that the investor has a choice among five investment strategies (**A**, **B**, **C**, **D** and **E**). Before estimating the historical investment performance by back-testing the different diversification strategies on the stock and/or commodity markets, we first outline the structure of each of the strategies considered (See Table 2).

Table 3 Diversification strategies

	Strategy A	Strategy B	Strategy C	Strategy D	Strategy E
NAM	✓		✓	✓	
EUR	✓		✓	✓	
APC	✓		✓	✓	
LAM	✓		✓	✓	
GCC	✓		✓	✓	
WRD					✓
WTI1		✓	✓		✓
GAS1		✓	✓		✓
SOY1		✓	✓		✓
SOL1		✓	✓		✓
SGR1		✓	✓		✓
WHT1		✓	✓		✓
CPR1		✓	✓		✓
GLD1		✓	✓		✓
WTI70		✓		✓	✓
GAS70		✓		✓	✓
SOY17		✓		✓	✓
SOL20		✓		✓	✓
SGR8		✓		✓	✓
WHT12		✓		✓	✓
CPR10		✓		✓	✓
GLD12		✓		✓	✓

- *Strategy A: Investing in regional stock indices*: This strategy consists of investing in the five regional stock indices (**NAM**, **EUR**, **APC**, **LAM** and **GCC**) only.
- *Strategy B: Investing in short- and long-term commodity futures*: This strategy consists of investing in short- (**WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**, **WHT1**, **CPR1** and **GLD1**) and long-term (**WTI70**, **GAS70**, **SOY17**, **SOL20**, **SGR8**, **WHT12**, **CPR10** and **GLD12**) commodity futures contracts.
- *Strategy C: Investing in regional stock indices & short-term commodity futures*: This strategy consists of investing in the five regional stock indices (**NAM**, **EUR**, **APC**, **LAM** and **GCC**), as well as short-term commodity futures contracts (**WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**, **WHT1**, **CPR1** and **GLD1**).
- *Strategy D: Investing in regional stock indices & long-term commodity futures*: This strategy consists of investing in the five regional stock indices (**NAM**, **EUR**, **APC**, **LAM** and **GCC**), as well as long-term commodity futures contracts (**WTI70**, **GAS70**, **SOY17**, **SOL20**, **SGR8**, **WHT12**, **CPR10** and **GLD12**).
- *Strategy E: Investing in a global stock index & short- and long-term commodity futures*: This strategy consists of investing in the global stock market index (**WRD**), as well as short-term (**WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**, **WHT1**, **CPR1** and **GLD1**) and long-term commodity futures contracts (**WTI70**, **GAS70**, **SOY17**, **SOL20**, **SGR8**, **WHT12**, **CPR10** and **GLD12**).

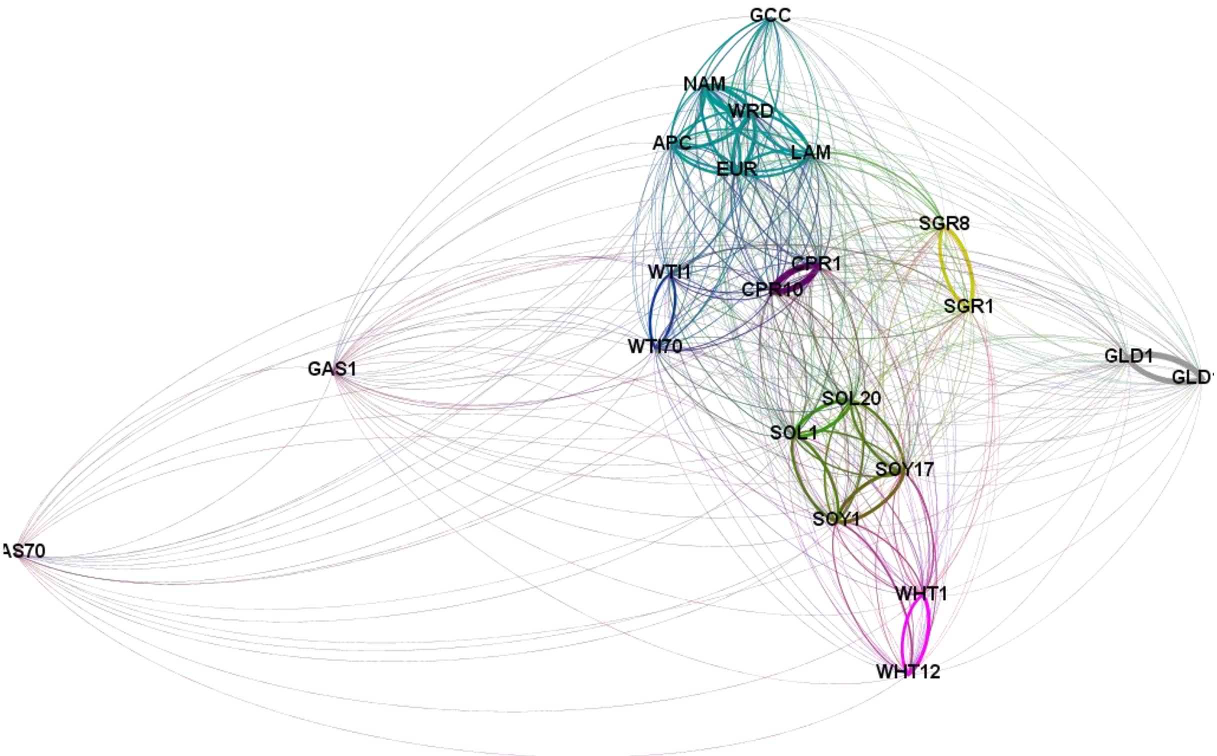
3. Results

In this section we analyze the connectedness among underlying assets for each of the five considered portfolios (section 4.1), then we analyze for each of the considered portfolios the different diversification strategies (section 4.2).

3.1. Connectedness analysis

Before analyzing the connectedness among the underlying assets for each of the portfolios considered, we start with an aggregated investigation of the connectedness among all the considered assets over the full sample period. Figure 1 summarizes directional connectedness among all the considered assets, based on the full sample average estimation. We notice that connectedness clustering is obvious. Indeed, the magnitude of directional connectedness is different depending on the type of the assets within each strategy, *i.e.*, that they tend to be grouped according to the main category to which each asset belongs (commodity or pure financial asset). For instance, the directional connectedness network plot reveals a relatively high level of integration among global and regional stock markets indices (**WRD**, **NAM**, **EUR**, **APC**, **LAM** and **GCC**), and between each of the short-term futures contracts and its long-term counterpart, except for **GAS1** and **GAS70**. However, **GAS1** and **GAS70** seem to be the most isolated markets, i.e. that they weakly affect (and are weakly affected by) other markets, which suggests that these commodities are potentially safe haven assets.

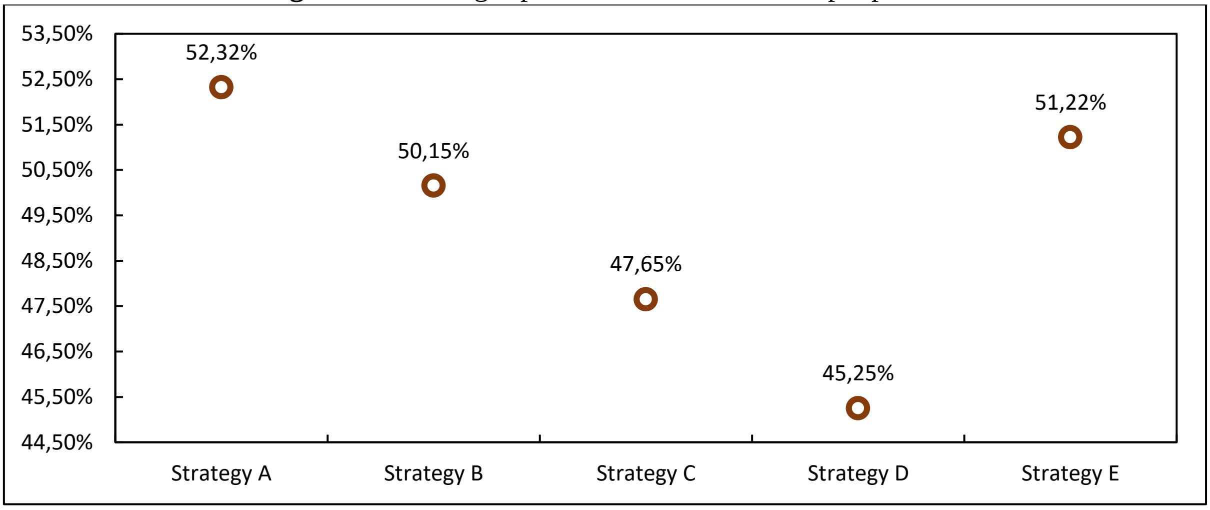
Fig. 1 Directional connectedness network plot – All underlying assets



Note: The Jacomy et al. (2014) algorithm has been used to determine the location of each node. This algorithm finds a steady state in which the forces of repulsion and attraction between nodes balance each other; while nodes repel each other, edges attract them to each other. The force of attraction of an edge is proportional to the average pairwise directional connectedness between the two nodes, which also defines the edge's thickness. The connectedness table used to plot this directional connectedness network is available from the authors upon request.

Tables 4 to 8 depict the total average connectedness indices (summarized in Figure 2) as well as their "input-output" decomposition for each of the investment strategies considered. Their (*i*,*j*)-th entries are the estimated contributions to the generalized forecast error variance components of market *i* coming from innovations in market *j*. The total connectedness index, reported in the south-east corner of each table, is the off-diagonal column sums (i.e., *contribution to others*) relative to the column sums including diagonals (i.e., *contribution to others including own*) expressed as a percentage. Spillovers from all other markets to a given market are reported in the last column.

Fig. 2 Total average spillovers over the full sample period



Note: Each point is an average for the corresponding strategy (see the x-axis)

Figure2 show that, for the full sample period, the total connectedness indices for the five selected strategies vary between 45.25 (**Strategy D**) and 52.32% (**Strategy A**), which suggests a relatively higher interdependency among pure financial markets on average and across the entire sample period, and a lower connectedness among regional stock markets and long-term commodity futures markets.

- *Connectedness among **Strategy A**'s panel of assets*

Results reported in Table 4 show that the total connectedness among the regional stock markets is relatively high (52.32%), which suggests a strong interdependence between these pure financial markets on average over the period studied. This result is consistent with what has been found by other previous empirical investigations (among others: Arouri et al., 2009; Diebold and Yilmaz, 2009; Arouri et al., 2011; Ben Amar et al. 2021; Bélaid et al. 2021).

Table 4 Connectedness Table - Strategy A

	NAM	EUR	APC	GCC	LAM	FROM
NAM	46.72	21.11	10.84	4.08	17.25	53.28
EUR	22.49	42.36	12.86	4.46	17.84	57.64
APC	21.28	19.52	38.83	4.76	15.61	61.17
GCC	9.34	9.47	8.32	63.85	9.02	36.15
LAM	18.13	18.62	11.42	5.2	46.63	53.37
TO	71.23	68.72	43.45	18.5	59.71	261.62
Inc.Own	117.96	111.08	82.27	82.35	106.34	**TCI**
NET	17.96	11.08	-17.73	-17.65	6.34	**52.32**

Notes: A TVP-VAR(0.99, 0.99) of order 1 was selected: the Bayesian Information Criterion was used to choose the lag order. See section 3.2. for abbreviations.

The pairwise connectedness to others (i) from each of the considered regional stock markets (j) tend to be grouped according to the region's level of development. Indeed, the magnitudes of the pairwise directional connectedness range from around 4% — spillovers from **GCC** to **NAM** (4.08%), **EUR** (4.46%) and **APC** (4.76%) — to almost 22% — spillovers from **NAM** to **EUR** (22.49%) and **APC** (21.28%), and from **EUR** to **NAM** (21.11%). Similarly, the total directional connectedness "*to others*" row suggests that the total spillovers to others from each of the considered regional markets tend to be grouped according to regional level of development. Indeed, the magnitudes of total directional connectedness "*to others*" vary between as low as 18.5% — total connectedness to others from **GCC** — to as high as 71.23% — total connectedness to others from **NAM**. The net connectedness, which is the difference between the "*to*" and "*from*" total directional connectedness indices, varies significantly across regions. While North American (**NAM**), European (**EUR**) and Latin America (**NAM**) stock markets tend, on average, to influence rather than be influenced by other regional stock markets (positive net connectedness indices), stock markets in the remaining regions (**APC** and **GCC**) tend to be influenced by rather than influence others (negative net connectedness indices). Of particular interest is that the **GCC** stock market is largely disconnected from other regional financial markets, as it does not influence nor is it influenced by other regional stock markets to any great extent. The diagonal of the pairwise connectedness matrix suggests that, with a contribution to own return forecast error variance of 63.85%, the **GCC** market is largely closed to itself.

- *Connectedness among **Strategy B**'s panel of assets*

Table 5 shows that the total connectedness among the short- and long-term commodity futures markets is relatively high (50.15%), which suggests a relatively high level of interdependency among commodity futures markets. Moreover, commodity futures markets seem to be clustered. Indeed, the magnitude of the pairwise directional connectedness to others (i) from each of the considered commodities (j) is different depending on the type and the maturity of the commodity futures, *i.e.*, that they tend to be grouped according to the main category to which each commodity belongs (energy, industrial & precious metals or agricultural).

[**Insert Table 5 about here**]

Except for natural gas futures markets, the pairwise directional connectedness between each of the short-term futures contracts and their respective long-term counterparts are homogenous and relatively high. A diagonal reading of Table 5 reveals that, with contribution to own forecast error variance ranging between 36 and 40%, copper, soybeans and soybean oil futures markets are on average much open to others than are the other commodity futures markets, which contribution to own return forecast variance could be as high as 80%. The natural gas futures markets seem to be largely insensitive to shocks on other energy, agricultural and metal commodities, and that spillovers to others from innovations to natural gas futures markets are relatively low as well and not very different for the considered short- and long-term maturities. Indeed, with contributions to own return forecast error variance of about 80%, the natural gas futures markets are, on average, the least open to other commodity markets. Furthermore, agricultural futures commodity markets are influenced more by other agricultural commodities, but they appear to be broadly insensitive to shocks on non-agricultural commodity markets. Indeed, agriculture corps are affected by each other since they substitute each other and compete for resources, such as fertilizers, water, and land (Baumeister and Kilian, 2014; Bastianin et al. 2014). These results support the thesis that commodity markets are potentially segmented, which is in line with the conclusions of Hammoudeh et al. (2010) and Gardebroek et al. (2016), and that GAS is potentially a safe haven asset, which is in line with the analysis of Ben Amar et al. (2022).

Net connectedness, NCI, which is computed as the difference between "*contribution to others*" and "*from others*" total directional connectedness, indicates whether a market is a net transmitter ($\text{NCI} > 0$) or a net receiver ($\text{NCI} < 0$) of volatility. We notice that **WTI**, **GAS**, **SGR** and **WHT** are, on average over the entire period and for all the considered nearest-to-maturities, net volatility receivers from all other commodities, while **SOY**, **SOL** and **CPR** are net volatility transmitters, which is in part consistent with the results of Zhang and Wei (2010) and Ahmadi et al. (2016). More interestingly, our results reveal that GLD is net transmitter of volatility in the short-term, but a net receiver in the long-term.

- *Connectedness among **Strategy C**'s panel of assets*

The total connectedness among regional stock markets and short-term commodity futures markets is about 47% (See Table 6). The results also indicate that the intensity of "*contribution to others*" from each of the markets considered differs substantially according to the type of market, i.e., the markets tend to be grouped according to the main category (commodity or stock market) to which they belong.

[**Insert Table 6 about here**]

While spillovers to others from regional stock markets (**NAM**, **EUR**, **APC**, **GCC** and **LAM**) range between 29% for the **GCC** market and 90% for the **EUR** market, spillovers to others from commodity short-term futures markets (**WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**, **WHT1**, **CPR1** and **GLD1**) range from 9% for the **GAS1** market to 54% for the **SOL1** market. This result supports that financial markets are potentially segmented, which is consistent with the results of Belke and Dubova (2018) and Hachicha et al. (2022). For instance, regional stock markets (**NAM**, **EUR**, **APC**, **GCC** and **LAM**) have a strong influence on each other, but short-term commodity futures markets appear to be poorly affected by shocks to non-commodity markets (and vice versa). Indeed, we note that commodity futures markets are hardly influenced by developments in regional stock markets, and that spillovers to regional stock markets from shocks to commodity markets are quite low as well. More specifically, innovations to **WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**,**WHT1**, **CPR1** and **GLD1** returns are respectively responsible for 17.26%, 3.41%, 6.73%, 13.38%, 7.89%, 4.31%, 26.08%, and 7.92% of the error variance in forecasting 10-days-ahead of all regional stock markets' returns, and the error variance in forecasting 10-days-ahead **WTI1**, **GAS1**, **SOY1**, **SOL1**, **SGR1**,**WHT1**, **CPR1** and **GLD1** returns coming from innovations to other non-commodity markets are 22.75%, 7.72%, 9.99%, 16.86%, 14.91%, 5.74%, 32.08% and 16.07%, respectively. This result implies that commodity futures, especially **GAS1**, **SOY1** and **WHT1**, seem to potentially be safe havens. Furthermore, while spillovers to others (j) from each of the markets considered (i) seem also to support that stock and commodity markets are highly segmented, they nevertheless show that purely financial markets (**NAM**, **EUR**, **APC**, **GCC** and **LAM**) are relatively much more integrated than commodity markets. In addition, shocks to short-term commodity futures markets have relatively small effects on regional stock markets. Likewise, we notice that the magnitude of spillovers to all other markets from each of the commodity and stock markets are widely consistent within each of the two asset classes. From the net connectedness indices, we note that all commodity markets, except for **SOL1**, are net receivers of volatility, suggesting that their dynamics are relatively more influenced by (than influencing) the dynamics of regional stock markets. However, the regional stock markets **NAM**, **EUR** and **LAM** are the main net volatility transmitters to all other markets.

- *Connectedness among **Strategy D**'s panel of assets*

Table 7 depicts the connectedness metrics among regional stock markets and long-term commodity futures markets. We find that the connectedness structure is very similar to that of strategy C. Indeed, the average total connectedness index over the sample period is 45%. This level, which is close to, but slightly lower than, that of strategy C, indicates that comovements within strategy's D panel of assets are rather moderate. Also, connectedness is clustering is obvious *i.e.*, markets tend to be clustered according to the main category (commodity or stock market) to which they belong.

[**Insert Table 7 about here**]

Commodity markets appear to be relatively less open to other markets than stock markets. Indeed, we notice that regional stock markets have a relatively strong influence on each other, but that long-term commodity futures markets are hardly influenced by what is happening in the regional stock markets, and that shocks to long-term commodity markets have a weak influence on regional stock markets. This result supports again that stock and commodity markets are potentially weakly integrated, but that regional stock markets are relatively much more integrated than commodity futures markets, which is in line with the findings of Belke and Dubova (2018), Ben Amar et al. (2021) and Hachicha et al. (2022). Moreover, connectedness results suggest that commodity futures, especially **GAS70**, **WHT12** and **GLD12**, seem to potentially be safe havens assets. From the net connectedness indices, we note that all long-term commodity futures markets, except for **SOL20**, are net receivers of volatility, suggesting that their dynamics are relatively more influenced by (than influencing) the dynamics of regional stock markets. However, the regional stock markets **EUR**, **LAM** and **NAM** are the most important net volatility transmitters to all other markets.

- *Connectedness among **Strategy E**'s panel of assets*

Table 8 reports the connectedness metrics among the global stock market (**WRD**) and short- and long-term commodity futures markets. The total connectedness among the underlying assets of this strategy is about 51%.

[**Insert Table 8 about here**]

According to the results, each of the pairs **CPR1** and **CPR10**, and **GLD1** and **GLD12** are relatively tightly connected to each other. With contributions to own return forecast error variance of 33.76%, 34.07, 36.15% and 36.35% respectively, **CPR10**, **CPR1**, **SOL1** and **SOY1** are the most open to other markets. On the other side, with contributions to own return forecast error variance of 81.11% and 78.11% respectively, **GAS1** and **GAS70** are the least open to other markets: only 21.89% and 18.89%, respectively, of **GAS1** and **GAS70** dynamics is due to network connectedness. This result suggests that **GAS1** and **GAS70** are potentially (**i**) save havens assets for global investors and, as will be seen later when we get to the section of portfolio analysis, (**ii**) good portfolio diversifiers. From the net connectedness indices, we note that the largest net receivers of shocks from the system are **SGR8**, **GAS1** and **GAS70**, and the largest net transmitters of shocks to the system are **SOL1**, **SOY1** and **CPR10**.

Table 5 Connectedness Table - Strategy B

	WTI1	WTI70	GAS1	GAS70	SOY1	SOY17	SOL1	SOL20	SGR1	SGR8	WHT1	WHT12	CPR1	CPR10	GLD1	GLD12	FROM
WTI1	49.54	14.13	1.18	0.79	2.74	2.1	4.76	3.21	4.68	2.13	1.5	1.13	5.09	5.1	1.31	0.61	50.46
WTI70	15.36	51.8	1.48	0.57	2.3	2.71	3.35	3.34	2.62	2.18	2.33	1.31	3.47	3.72	2.28	1.18	48.2
GAS1	2.16	2.57	78.66	1.54	0.87	0.83	1.38	1.45	1.05	1.16	1.16	0.84	1.99	1.97	1.14	1.22	21.34
GAS70	1.01	0.82	1.71	81.91	1.27	1.15	1.8	1.36	0.74	0.48	1.21	0.66	1.45	1.43	1.22	1.77	18.09
SOY1	2.07	1.61	0.32	0.57	36.22	14.37	13.43	7.75	2.08	2.36	6.6	5.44	2.19	2.36	1.68	0.96	63.78
SOY17	1.45	1.57	0.44	0.68	15.76	41.22	7.8	10.99	2.45	3.64	4.79	2.7	1.77	1.92	1.63	1.19	58.78
SOL1	3.26	2.18	0.67	0.42	13.53	7	37.39	16.38	1.92	1.94	3.04	3.26	3	3.12	1.84	1.03	62.61
SOL20	2.28	2.15	0.76	0.43	8.14	10.67	17.65	40.34	1.89	2.1	2.29	2.06	2.93	3.12	1.78	1.4	59.66
SGR1	4.6	2.83	0.86	0.57	3	3.39	3.13	2.62	48.19	17.76	3.1	2.36	2.08	2.04	2.1	1.38	51.81
SGR8	2.15	2.36	1.29	0.36	3.27	4.57	3.16	3.21	17.91	49.74	1.63	1.48	2.94	3.15	1.51	1.28	50.26
WHT1	2.05	2.34	0.62	0.62	8.15	5.35	3.88	2.81	2.76	1.43	47.22	17.93	1.22	1.23	1.55	0.86	52.78
WHT12	1.29	0.92	0.54	0.29	7.28	3.34	4.34	2.31	2.35	1.2	19.62	52.38	0.95	0.94	1.66	0.58	47.62
CPR1	3.68	2.44	1.06	0.65	2.46	1.8	2.83	2.9	1.45	1.65	1.21	1.06	36.87	35.71	2.61	1.62	63.13
CPR10	3.65	2.56	1.14	0.66	2.48	1.92	2.98	3.07	1.42	1.71	1.18	1.02	35.39	36.7	2.53	1.59	63.3
GLD1	1.51	1.86	0.77	0.54	1.55	1.28	2.24	1.59	1.39	1.13	1.25	1.56	2.77	2.87	52.87	24.81	47.13
GLD12	0.84	1.27	0.71	1.28	1.11	1.41	1.55	1.53	0.8	0.84	1.06	0.71	1.81	1.89	26.62	56.56	43.44
TO	47.35	41.61	13.56	9.98	73.92	61.91	74.27	64.51	45.49	41.7	51.98	43.53	69.04	70.58	51.47	41.48	802.39
Inc.Own	96.89	93.42	92.22	91.89	110.14	103.14	111.67	104.85	93.68	91.44	99.2	95.91	105.91	107.28	104.34	98.03	**TCI**
NET	-3.11	-6.58	-7.78	-8.11	10.14	3.14	11.67	4.85	-6.32	-8.56	-0.8	-4.09	5.91	7.28	4.34	-1.97	**50.15**

Notes: A TVP-VAR(0.99, 0.99) of order 1 was selected: the Bayesian Information Criterion was used to choose the lag order. See section 3.2. for abbreviations.

Table 6 Connectedness Table - Strategy C

	NAM	EUR	APC	GCC	LAM	WTI1	GAS1	SOY1	SOL1	SGR1	WHT1	CPR1	GLD1	FROM
NAM	40	17.92	9	3.08	14.06	3.27	0.55	0.89	2.41	1.16	0.51	5.62	1.54	60
EUR	18.44	34.21	10.69	3.55	14.13	3.17	0.73	1.29	2.82	1.42	1.34	6.42	1.78	65.79
APC	17.37	16.63	32.57	3.5	12.9	2.66	0.85	1.32	2.17	1.5	0.8	5.75	1.98	67.43
GCC	7.72	8.27	6.84	53.92	7.46	4.54	0.55	1.25	3.17	1.66	0.77	2.58	1.27	46.08
LAM	14.28	14.78	9.37	4.25	38.1	3.62	0.73	1.98	2.81	2.15	0.89	5.71	1.35	61.9
WTI1	5.05	4.89	3.41	3.64	5.76	54.63	0.97	2.77	5.31	4.82	1.63	5.59	1.53	45.37
GAS1	1.78	1.86	1.45	1.16	1.47	1.68	83.56	0.8	1.38	0.85	1	1.9	1.12	16.44
SOY1	1.65	2.18	1.75	1.29	3.12	3.22	0.53	50.9	18.64	2.71	9.18	3.17	1.66	49.1
SOL1	3.53	4.05	2.9	2.43	3.95	4.58	0.83	17.47	47.45	2.48	4.13	4.2	1.99	52.55
SGR1	2.65	3.74	2.32	2.18	4.02	5.68	0.7	3.55	3.47	62.96	4.2	2.49	2.03	37.04
WHT1	1.01	1.52	0.9	0.79	1.52	2.75	0.81	11.53	5.64	4.11	65.96	2.04	1.42	34.04
CPR1	7.72	9.29	6.07	1.54	7.46	4.87	1.03	3.14	3.8	1.88	1.75	47.9	3.55	52.1
GLD1	3.46	4.95	3.03	1.65	2.98	2.53	0.99	1.76	2.92	1.8	1.64	3.86	68.43	31.57
TO	84.67	90.08	57.73	29.05	78.82	42.57	9.27	47.75	54.54	26.53	27.85	49.33	21.21	619.4
Inc.Own	124.67	124.29	90.3	82.97	116.91	97.2	92.82	98.65	101.99	89.49	93.81	97.24	89.64	**TCI**
NET	24.67	24.29	-9.7	-17.03	16.91	-2.8	-7.18	-1.35	1.99	-10.51	-6.19	-2.76	-10.36	**47.65**

Notes: A TVP-VAR(0.99, 0.99) of order 1 was selected: the Bayesian Information Criterion was used to choose the lag order. See section 3.2. for abbreviations.

Table 7 Connectedness Table - Strategy D

	NAM	EUR	APC	GCC	LAM	WTI70	GAS70	SOY17	SOL20	SGR8	WHT12	CPR10	GLD12	FROM
NAM	39.94	17.15	9.64	4.69	12.37	2.31	0.87	0.94	1.76	2.31	1.02	6.08	0.91	60.06
EUR	16.78	34.42	13.53	5.41	11.12	2.5	0.84	0.9	2.1	4.64	1.28	5.68	0.81	65.58
APC	16.21	17.9	33.62	5.45	9.73	2.48	0.82	0.72	1.5	4.25	1.16	5.34	0.81	66.38
GCC	7.79	8.58	7.84	56.27	5.95	2.4	0.91	1.12	1.98	2.14	1.03	2.9	1.08	43.73
LAM	12.16	11.64	8.08	5.22	39.49	3.9	1.05	3.23	3.41	4.14	1.34	5.56	0.76	60.51
WTI70	3.58	4.97	4.59	3.54	6.77	59.3	0.86	3.25	4.19	2.1	1.71	4.13	1	40.7
GAS70	1.15	2.49	1.91	2.1	2.28	1.18	81.67	1.23	1.51	0.78	0.77	1.29	1.65	18.33
SOY17	1.62	1.82	1.5	1.85	5.34	2.48	1.17	56.4	16.07	3.98	3.76	2.7	1.29	43.6
SOL20	2.21	3.01	2.13	2.32	4.62	2.99	0.99	15.42	55.23	2.34	2.65	4.45	1.63	44.77
SGR8	4.25	7	5.85	2.79	6.94	2.17	0.89	4.06	3.02	57.13	1.58	3.4	0.9	42.87
WHT12	2	3	1.85	1.48	2.83	1.6	0.69	4.67	3.71	1.91	74.23	1.3	0.71	25.77
CPR10	8.27	8.82	6.39	3.12	7.46	3.27	1.27	2.35	4.05	2.55	1.2	49.19	2.06	50.81
GLD12	1.71	3.08	2.15	2.5	2.94	1.74	2	1.76	2.33	1.55	0.78	2.64	74.82	25.18
TO	77.74	89.47	65.47	40.46	78.36	29.02	12.38	39.65	45.63	32.71	18.28	45.49	13.61	588.27
Inc.Own	117.68	123.9	99.09	96.74	117.85	88.32	94.05	96.05	100.86	89.85	92.51	94.67	88.43	**TCI**
NET	17.68	23.9	-0.91	-3.26	17.85	-11.68	-5.95	-3.95	0.86	-10.15	-7.49	-5.33	-11.57	**45.25**

Notes: A TVP-VAR(0.99, 0.99) of order 1 was selected: the Bayesian Information Criterion was used to choose the lag order. See section 3.2. for abbreviations.

Table 8 Connectedness Table - Strategy E

	WRD	WTI1	WTI70	GAS1	GAS70	SOY1	SOY17	SOL1	SOL20	SGR1	SGR8	WHT1	WHT12	CPR1	CPR10	GLD1	GLD12	FROM
WRD	51.43	4.5	3.79	0.92	0.69	1.62	1.52	3.73	2.67	1.78	2.34	1.01	1.54	9.39	9.61	2.31	1.17	48.57
WTI1	4.81	46.61	13.56	1.16	0.82	2.5	1.99	4.71	3.2	4.47	1.98	1.49	1.11	4.83	4.88	1.31	0.56	53.39
WTI70	4.15	15.13	49.21	1.34	0.57	2.3	2.73	3.46	3.49	2.17	1.83	2.4	1.26	3.23	3.51	2.12	1.11	50.79
GAS1	1.96	2.1	2.41	78.11	1.51	0.78	0.72	1.4	1.35	0.93	0.86	1.12	0.78	1.89	1.8	1.08	1.19	21.89
GAS70	1	1.22	0.8	1.77	81.11	1.25	1.17	1.94	1.24	0.7	0.44	1.25	0.64	1.51	1.46	0.81	1.68	18.89
SOY1	1.17	2.04	1.62	0.29	0.49	36.35	14.37	13.17	7.62	2.02	2.18	6.66	5.46	2.14	2.32	1.2	0.89	63.65
SOY17	1.56	1.46	1.65	0.4	0.6	15.59	41.06	7.51	10.92	2.32	3.52	4.84	2.73	1.68	1.85	1.14	1.18	58.94
SOL1	3.15	3.3	2.16	0.6	0.42	13	6.8	36.15	15.95	1.89	1.8	3	3.32	2.93	3.07	1.49	0.97	63.85
SOL20	2.43	2.32	2.25	0.69	0.41	7.87	10.51	17.19	39.55	1.81	1.89	2.28	2.19	2.8	3.01	1.45	1.35	60.45
SGR1	2.93	4.71	2.42	0.62	0.48	2.89	3.31	3.16	2.69	47.41	17.02	3.23	2.31	2	1.93	1.71	1.18	52.59
SGR8	4.38	2.14	1.83	0.93	0.33	2.92	4.26	3.03	2.92	17.16	48.98	1.68	1.36	2.82	3.04	1.19	1.03	51.02
WHT1	0.87	2.04	2.48	0.62	0.59	8.15	5.36	3.94	2.85	2.84	1.47	46.44	17.67	1.21	1.22	1.42	0.85	53.56
WHT12	1.58	1.2	0.9	0.53	0.26	7.19	3.42	4.41	2.38	2.19	1.17	19.29	51.53	0.9	0.88	1.6	0.56	48.47
CPR1	7.11	3.48	2.3	0.92	0.63	2.34	1.75	2.86	2.74	1.26	1.46	1.27	1.14	34.07	32.95	2.25	1.46	65.93
CPR10	7.33	3.46	2.44	0.93	0.65	2.35	1.86	2.99	2.9	1.24	1.55	1.22	1.08	32.53	33.76	2.22	1.5	66.24
GLD1	2.6	1.63	1.93	0.73	0.54	1.6	1.32	2.79	1.75	1.35	1.1	1.25	1.39	2.55	2.65	51.11	23.71	48.89
GLD12	0.97	0.9	1.39	0.67	1.28	1.12	1.48	1.89	1.66	0.77	0.81	1.04	0.56	1.67	1.76	25.59	56.44	43.56
TO	47.99	51.63	43.94	13.12	10.27	73.47	62.58	78.19	66.32	44.9	41.41	53.02	44.54	74.08	75.93	48.89	40.39	870.68
Inc.Own	99.43	98.25	93.14	91.24	91.38	109.82	103.64	114.35	105.88	92.31	90.38	99.46	96.07	108.15	109.69	100	96.82	**TCI**
NET	-0.57	-1.75	-6.86	-8.76	-8.62	9.82	3.64	14.35	5.88	-7.69	-9.62	-0.54	-3.93	8.15	9.69	0	-3.18	**51.22**

Notes: A TVP-VAR(0.99, 0.99) of order 1 was selected: the Bayesian Information Criterion was used to choose the lag order. See section 3.2. for abbreviations.

3.2. Dynamic portfolio analysis

In this section we examine the five portfolio investment strategies using the three approaches described in section 3.1.ii, namely (i) the traditional Markowitz (1959) minimum-variance approach (MIN-VAR), (ii) the Christoffersen et al. (2014) minimum-correlation approach (MIN-COR) and (iii) the Broadstock et al. (2022) minimum-connectedness approach (MIN-CON). It is perhaps useful to recall that with respect to the three competing portfolio construction approaches we consider in this paper, the MIN-VAR approach inherently minimizes the volatility of the portfolio, while the MIN-COR approach minimizes the correlations among the underlying assets. As for the MIN-CON approach, it minimizes the pairwise connectedness, and thus bilateral spillovers, among the underlying assets.

- *Strategy A*

We begin by analysing strategy A, which is composed of assets **NAM**, **EUR**, **LAM**, **APC** and **GCC**.

Table 9: Average portfolio weights during stress and stress-free periods - Strategy A

Period	Approach	NAM	EUR	APC	GCC	LAM
Pre-COVID-19 stress-free period	MIN-VAR	0.0756	0.099	0.5405	0.2848	0
	MIN-COR	0.1462	0.1212	0.2307	0.3305	0.1713
	MIN-CON	0.1086	0.1419	0.1817	0.3518	0.2161
COVID-19 period (Q1 2020)	MIN-VAR	0.1121	0.1379	0.5483	0.2018	0
	MIN-COR	0.1556	0.1551	0.2305	0.3463	0.1125
	MIN-CON	0.0677	0.1957	0.2118	0.37	0.1548
COVID-19 period (Apr 1, 2020 to Feb 23, 2022)	MIN-VAR	0.0867	0.0521	0.4679	0.3933	0
	MIN-COR	0.1898	0.0939	0.2184	0.3681	0.1298
	MIN-CON	0.1791	0.0816	0.2036	0.3806	0.1551
Russia-Ukraine war period	MIN-VAR	0.1019	0.0461	0.3954	0.4409	0.0157
	MIN-COR	0.2121	0.1286	0.1533	0.353	0.1529
	MIN-CON	0.1573	0.1399	0.1453	0.3601	0.1975

Figure 3 illustrates the dynamic portfolio compositions for investment strategy A under the three approaches – MIN-VAR, MIN-COR and MIN-CON, and Table 9 summarizes the average weights during stress and stress-free periods. A simple visual inspection reveals that the composition of MIN-VAR differs markedly from that of MIN-COR or MIN-CON, while MIN-COR and MIN-CON share fairly similar structures. More interestingly, we find that the structure of the portfolios under the three approaches changed substantially once following the COVID-19 crisis and a second time following the onset of the Russian-Ukrainian war. During the quiet period, while the MIN-VAR method gives on average more weight to **APC**, the MIN-COR and MIN-CON methods give on average more weight to **GCC**. Indeed, the portfolio structure suggested by the MIN-VAR method before the COVID-19 crisis (stress-free period) assigns relatively more weight to **APC** (about 54.05% on average) and **GCC** (about 28.48%), while the weight of **NAM** (about 7.56%), **EUR** (about 10%) and **LAM** (0%) are quite small. Over the same stress-free period, the MIN-COR and MIN-CON methods suggest (on average) weights of 23.07% and 18.17% respectively for **APC**, 33.05% and 35.18% respectively for **GCC**, 14.62% and 10.86% respectively for **NAM**, 12.12% and 14.19% respectively for **EUR**, and 17.13% and 21.61% respectively for **LAM**.

[**Insert Figure 3 about here**]

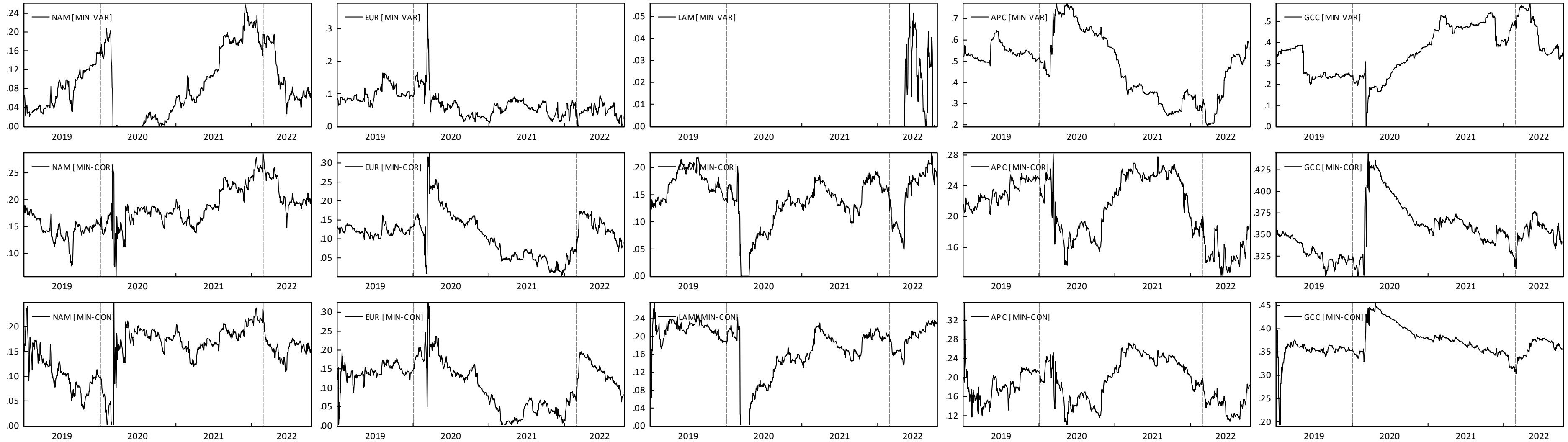
While Table 9 indicates that, on average, portfolio composition did not change too much during the COVID-19 period, Figure 3 shows that the dynamic portfolio structures have shifted significantly with the COVID-19 crisis. Indeed, over the period from January 1st, 2020 to February 23rd, 2022, the MIN-VAR method suggests an increasing trend in the weight of **NAM** – after a drastic drop from the end of February 2020 to the end of July 2020 – and of **GCC**, and a decreasing trend in the weight of **APC** and **EUR**. As for **LAM**, its weight in the MIN-VAR portfolio is 0% throughout the COVID-19 period. Over the same COVID-19 period, the MIN-COR and MIN-CON methods suggest a downward trend in the weights of **GCC** and **EUR**, after an increase towards the end of the first quarter of 2020. As for the weights of **NAM** and **LAM**, they experienced a drop towards the end of the first quarter of 2020, before gradually increasing thereafter to return to their pre-crisis levels. Regarding **APC**, its weight also dropped towards the end of the first quarter of 2020 to less than 15%. Then it gradually increased until the second quarter of 2021 to reach about 25%, before gradually decreasing thereafter over the rest of the observation period.

Over the period of the war in Ukraine, which began on February 24th, 2022, all three methods suggest that **GCC** is assigned the highest weight: 44.09% on average according to the MIN-VAR method, 35.3% on average according to the MIN-COR method, and 36.01% on average according to the MIN-CON method. We also note that, following the war in Ukraine, the weight of **LAM** according to the MIN-COR and MIN-CON methods shows an upward trend, and according to the MIN-VAR method increases from 0% to about 1.57% on average over the war period.

[**Insert Figure 4 about here**]

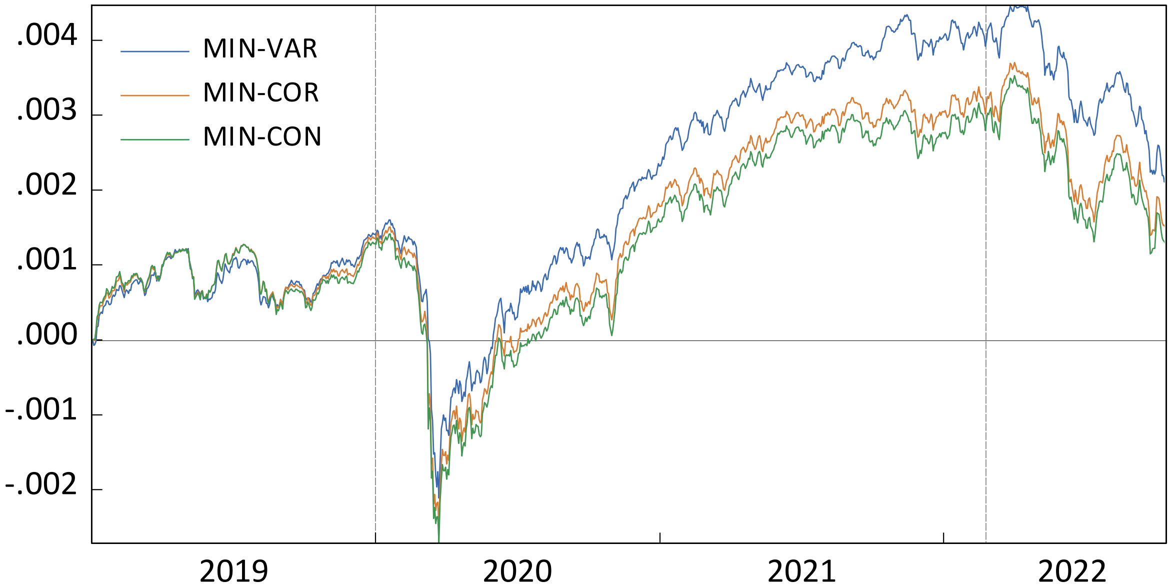
Figure 4 plots the performance, in terms of cumulative return, of the three alternative portfolio structures for Strategy A's panel of assets. While the plot shows that the minimum-variance (MIN-VAR), minimum-correlation (MIN-COR) and minimum-connectedness (MIN-CON) methods have visibly equivalent performance until early 2020, which corresponds to a stress-free period, the minimum-variance (MIN-VAR) method provides investors with a relatively higher performance from the beginning of the second quarter of 2020, concomitant with the COVID-19 crisis. Furthermore, all three approaches share the same underlying dynamics, including a significant decline in portfolio performance in the first quarter of 2020, followed by a pattern of sustained growth through February 2022, before dropping in the wake of the onset of the war in Ukraine.

Fig. 3 Strategy A - Weights



Notes: Strategy A consists of investing in regional stock indices. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 4 Strategy A - Cumulative returns



Notes: MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

- *Strategy B*

We now analyse strategy B, which is composed of assets **WTI1**, **WTI70**, **GAS1**, **GAS70**, **WHT1**, **WHT12**, **SOY1**, **SOY17**, **SOL1**, **SOL20**, **SGR1**, **SGR8**, **CPR1**, **CPR10**, **GLD1** and **GLD12**.

Table 10: Average portfolio weights during stress and stress-free periods - Strategy B

Period	Approach	WTI1	WTI70	GAS1	GAS70	SOY1	SOY17	SOL1	SOL20	SGR1	SGR8	WHT1	WHT12	CPR1	CPR10	GLD1	GLD12
Pre-COVID-19 stress-free period	MIN-VAR	0.000	0.055	0.014	0.098	0.000	0.197	0.000	0.049	0.002	0.059	0.000	0.102	0.000	0.242	0.027	0.154
	MIN-COR	0.019	0.071	0.159	0.133	0.015	0.005	0.007	0.054	0.081	0.059	0.031	0.105	0.036	0.086	0.066	0.075
	MIN-CON	0.053	0.068	0.107	0.111	0.014	0.044	0.035	0.053	0.068	0.045	0.062	0.079	0.036	0.099	0.057	0.071
COVID-19 period (Q1 2020)	MIN-VAR	0.000	0.034	0.007	0.072	0.001	0.243	0.000	0.028	0.003	0.055	0.000	0.102	0.000	0.318	0.037	0.101
	MIN-COR	0.004	0.049	0.108	0.132	0.005	0.027	0.004	0.057	0.083	0.034	0.051	0.085	0.001	0.203	0.081	0.078
	MIN-CON	0.035	0.087	0.104	0.116	0.014	0.051	0.015	0.076	0.070	0.063	0.064	0.073	0.041	0.057	0.055	0.078
COVID-19 period (Apr 1, 2020 to Feb 23, 2022)	MIN-VAR	0.000	0.067	0.005	0.106	0.022	0.218	0.000	0.031	0.000	0.077	0.000	0.122	0.004	0.193	0.018	0.137
	MIN-COR	0.070	0.025	0.106	0.163	0.024	0.075	0.009	0.021	0.026	0.047	0.043	0.053	0.067	0.145	0.016	0.112
	MIN-CON	0.060	0.055	0.108	0.116	0.024	0.063	0.030	0.047	0.046	0.068	0.038	0.072	0.112	0.027	0.066	0.068
Russia-Ukraine war period	MIN-VAR	0.000	0.084	0.002	0.081	0.003	0.123	0.001	0.011	0.003	0.215	0.000	0.053	0.006	0.115	0.054	0.249
	MIN-COR	0.012	0.061	0.123	0.161	0.044	0.010	0.019	0.066	0.034	0.064	0.031	0.104	0.162	0.000	0.014	0.097
	MIN-CON	0.036	0.071	0.114	0.120	0.039	0.020	0.034	0.051	0.024	0.081	0.053	0.072	0.160	0.000	0.027	0.098

Figures 5.a, 5.b and 5.c plot the dynamic portfolio weights for investment strategy B under the three approaches – MIN-VAR, MIN-COR and MIN-CON, and Table 10 summarizes the average weights during stress and stress-free periods. The results show that the structure of the MIN-VAR portfolio differs significantly from that of the MIN-COR and MIN-CON portfolios, while the MIN-COR and MIN-CON portfolios share fairly similar structures. We also find that the structure of the portfolio under the three approaches (i) is not stable over time, and (ii) has experienced two major changes: it shifted once following the COVID-19 crisis and a second time following the onset of the Russian-Ukrainian war. During the pre-COVID-19 stress-free period, the MIN-VAR portfolio gives on average more weight to **CPR10**, **SOY17**, **GLD12** and **WHT12**, while the MIN-COR and MIN-CON portfolio give on average more weight to **GAS1** and **GAS70**. Indeed, the portfolio structure suggested by the

MIN-VAR method before the COVID-19 crisis (stress-free period) assigns relatively more weight to **CPR10** (about 24.2% on average), SOY17 (about 19.7%), GLD12 (about 15.4%) and **WHT12** (about 10.2%), while the weight of **WTI1**, **SOY1**, **SOL1**, **SGR1**, **WHT1**, **CPR1**, **GAS1** and **GLD1** are null or almost null. Thus, a very important observation is that the MIN-VAR strategy gives much more weight to long-term commodity futures and relatively too little weight to short-term commodity futures. Over the same pre-COVID-19 stress-free period, the MIN-COR and MIN-CON methods suggest (on average) weights of 15.9% and 10.7% respectively for **GAS1**, and 13.3% and 11.1% respectively for **GAS70**. This result is quite expected as the natural gas market is the most disconnected and decoupled from other commodity markets.

[**Insert Figures 5.a, 5.b and 5.c about here**]

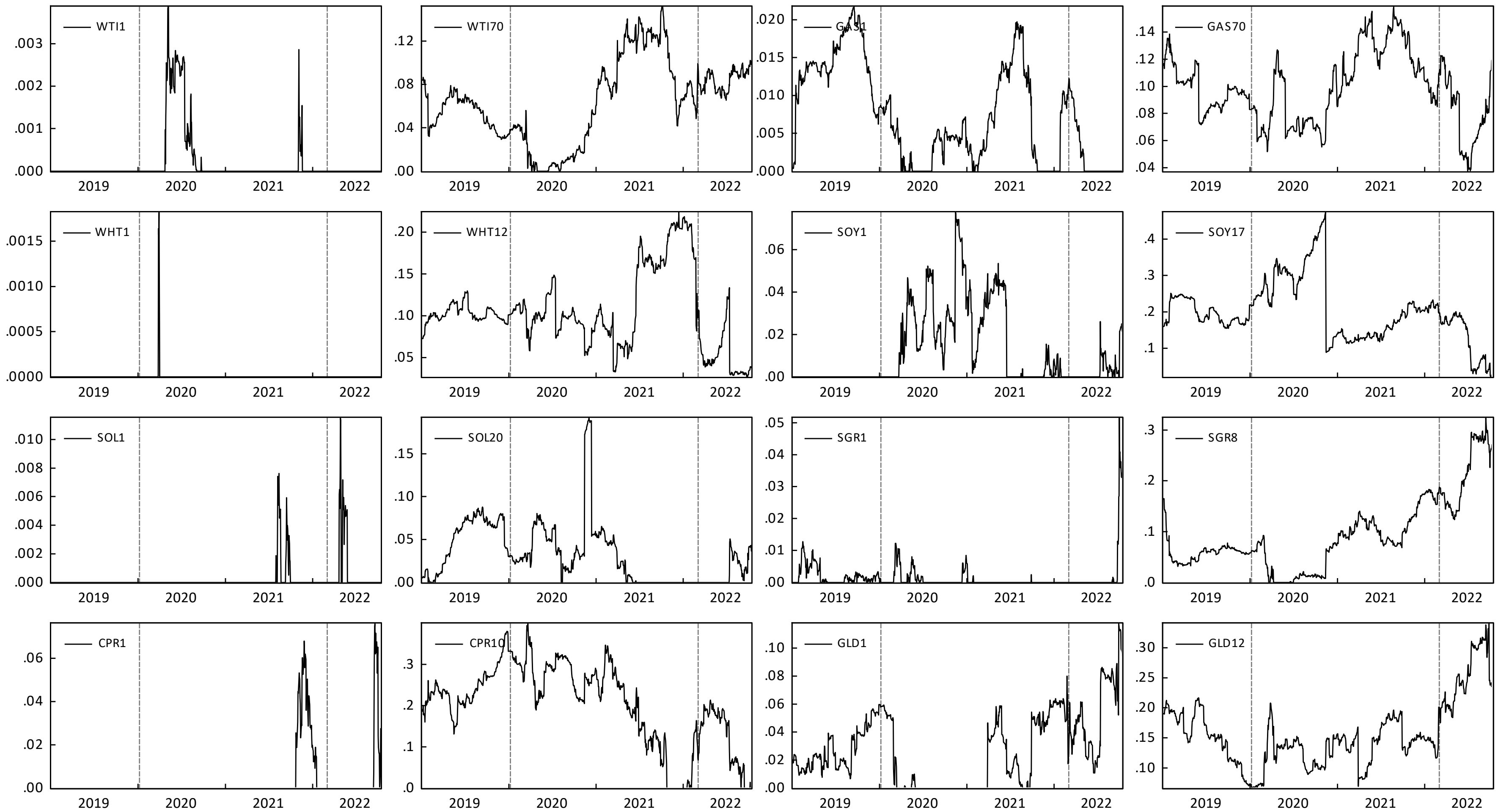
Figures 5.a, 5.b and 5.c shows that the dynamic portfolio weights have shifted significantly during the COVID-19 crisis period. In particular, during the first quarter of 2020, we observe a temporary increase in the weight of WTI1 in the portfolio under the MIN-VAR and the MIN-COR approaches, and a temporary decrease under the MIN-CON approach. Moreover, over the same period, the MIN-VAR method suggests an increasing trend in the weights of **GAS70**, **SGR8** and **WTI70**, and a decreasing trend in the weight of **CPR10**. The weights of the other commodities do not show any particular trends, but (**i**) successions of sharp increases and decreases (**WTI1**, **GAS1**, **SOY17**, **SOL1**, **SOL20**, **CPR1**, **SOY1**, **GLD1**), (**ii**) or largely stable weights (**WHT12**, **GLD12**). As for **WHT1** and **SGR1**, their weight in the MIN-VAR portfolio is almost 0% throughout the COVID-19 period. Over the same COVID-19 crisis period, the MIN-COR portfolio assigns the highest weights to assets **CPR10** (20.3% on average), **GAS70** (13.2% on average) and **GAS1** (10.8% on average), and the lowest weights to assets **CPR1** (0.1% on average), **SOL1** (0.4% on average), **WTI1** (0.4% on average) and **SOY1** (0.5% on average). As for MIN-CON portfolio approach, it assigns the highest weights to **GAS70** (11.6% on average), **GAS1** (10.4% on average) and **WTI70** (8.7% on average), and the lowest weights to assets **SOY1** (1.4% on average) and **SOL1** (1.5% on average). Moreover, the MIN-CON method suggests an upward trend in the weight of **CPR1** and **WHT12** during the year 2021, after a decrease during the year 2020. Regarding **WHT1** and **CPR10**, their weights dropped to 0% towards the mid-2021. Also, we find that the weights suggested by the MIN-CON method are broadly more stable over time than those suggested by MIN-COR and MIN-VAR methods.

Over the Russia-Ukraine war period, the MIN-COR and MIN-CON portfolios share fairly similar weights structure. Both of them assign the highest weights to **CPR1** (16.2 and 16% on average, respectively), **GAS70** (16.1 and 12% on average, respectively) and **GAS1** (12.3 and 11.4% on average, respectively), and the lowest level to **CPR10** (0%). On the other hand, the MIN-VAR portfolio approach assigns the highest weights to **GLD12** (24.9% on average), **SGR8** (21.5%), **SOY17** (12.3%) and **CPR10** (11.5%), and the lowest weights to **WTI1** (0%), **WHT1** (0%), **SOL1** (0.1%), **GAS1** (0.2%), **SOY1** (0.3%), **SGR1** (0.3%) and **CPR1** (0.6%). In other words, the MIN-VAR approach assigns the highest weights to long-term commodity futures, and the lowest weights to short-term commodity futures. A simple visual inspection reveals that the outbreak of war in Ukraine was a major shock to market participants who, according to all three approaches, were called upon to make sudden changes to their portfolio structure. Indeed, we note that, following the war in Ukraine, the MIN-CON method show that the weights of **WTI70**, **WHT1**, **SOL1**, **GAS1**, **G1S70**, and **GLD12** jumped up suddenly, while in parallel the weights of **WTI1**, **WHT12**, **SOY1**, **SOY17**, **CPR1** and **GLD1** jumped down suddenly. These findings are largely confirmed by the other two approaches.

[**Insert Figure 6 about here**]

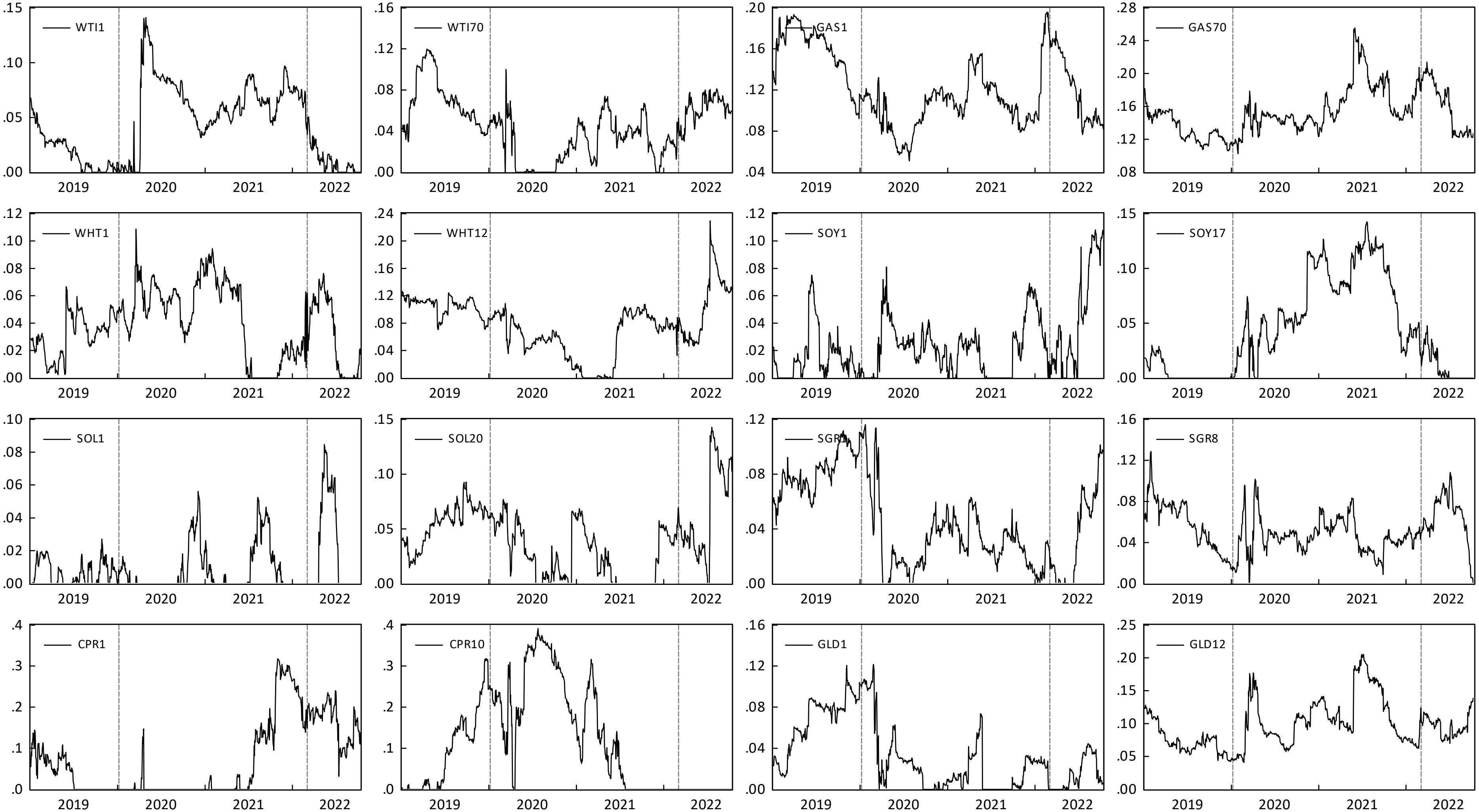
Figure 6 plots the performance, in terms of cumulative return, of the three alternative portfolio structures for Strategy B's panel of commodity futures. While the plot shows that the MIN-VAR, MIN-COR and MIN-CON methods record largely equivalent performances until early 2020, which corresponds to a stress-free period, the MIN-VAR method provides investors with the relatively lowest performance from the mid-2020, while the MIN-COR method provides them with a slightly highest performance. Furthermore, all three approaches share the same underlying dynamics, including a significant decline in portfolio performance in the first quarter of 2020, followed by a pattern of sustained growth through February 2022, before dropping in the wake of the onset of the war in Ukraine.

Fig. 5.a. Strategy B - Weights [Minimum-variance portfolio]



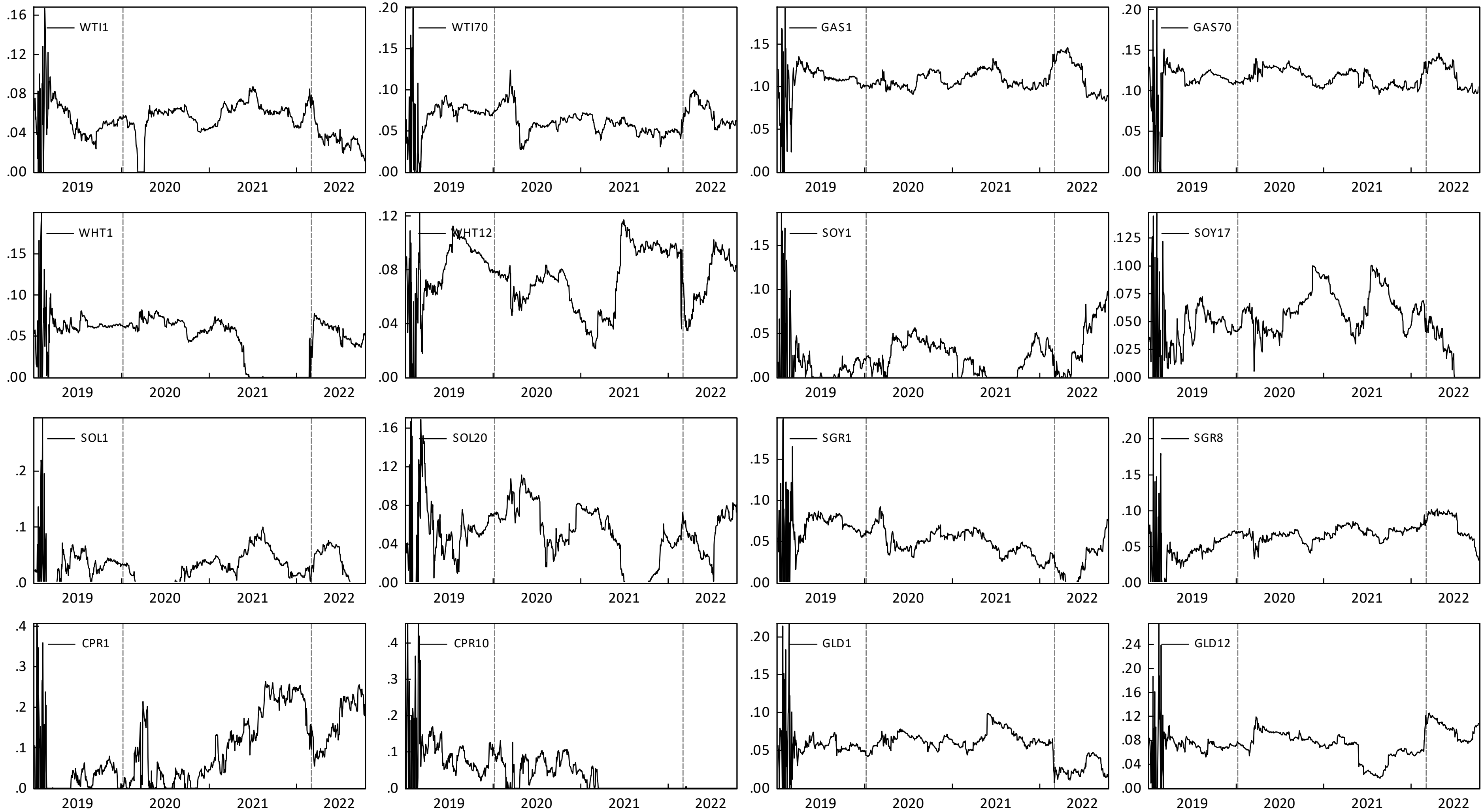
Notes: Strategy B consists of investing in short-run and long-run commodity futures. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS 1 and GAS 70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 5.b. Strategy B - Weights [Minimum correlation portfolio]



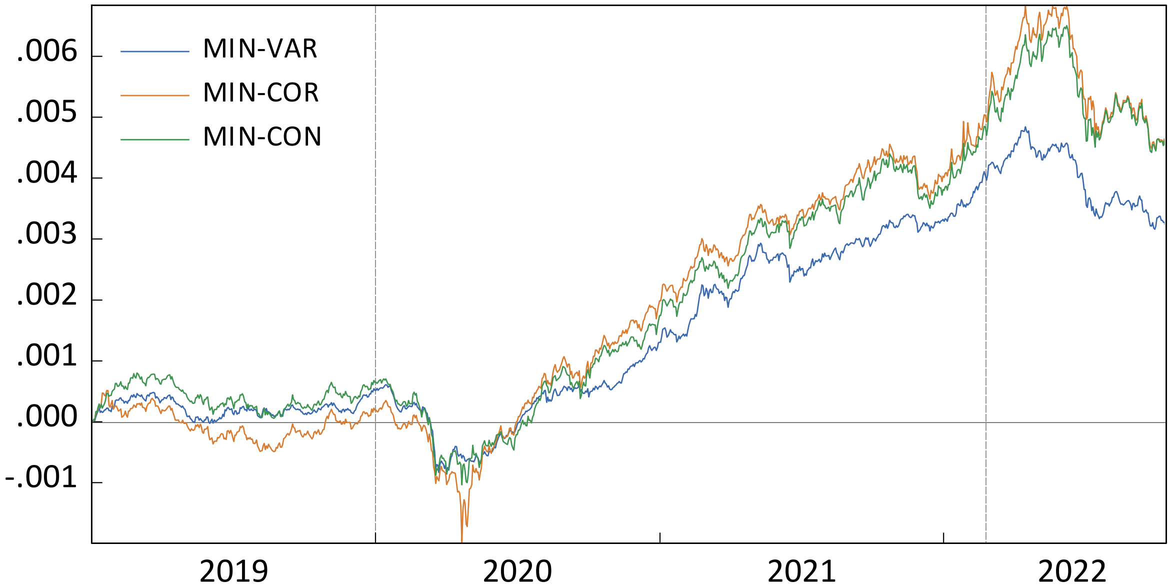
Notes: Strategy B consists of investing in short-run and long-run commodity futures. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS 1 and GAS 70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 5.c Strategy B - Weights [Minimum connectedness portfolio]



Notes: Strategy B consists of investing in short-run and long-run commodity futures. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS 1 and GAS 70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 6 Strategy B - Cumulative returns



Notes: MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

- *Strategy C*

We now analyse diversification strategy C, which is composed of regional indices (**NAM**, **EUR**, **LAM**, **APC**, **GCC**) and short-term commodity futures contracts (**WTI1**, **GAS1**, **WHT1**, **SOY1**, **SOL1**, **SGR1**, **CPR1**, and **GLD1**).

Table 11: Average portfolio weights during stress and stress-free periods - Strategy C

Period	Approach	NAM	EUR	APC	GCC	LAM	WTI1	GAS1	SOY1	SOL1	SGR1	WHT1	CPR1	GLD1
Pre-COVID-19 stress-free period	MIN-VAR	0.1056	0.0367	0.2246	0.1862	0	0	0.0208	0.0535	0.0179	0.0303	0.0288	0.0232	0.2725
	MIN-COR	0.0831	0.0112	0.0501	0.111	0.0008	0.053	0.1757	0.0152	0.0483	0.1061	0.1233	0.0599	0.1621
	MIN-CON	0.0287	0.0077	0.0652	0.102	0.045	0.0819	0.1355	0.0317	0.0822	0.1084	0.1154	0.0807	0.1155
COVID-19 period (Q1 2020)	MIN-VAR	0.1278	0.0293	0.2018	0.1428	0	0	0.016	0.0922	0.0031	0.0461	0.0285	0.0664	0.2458
	MIN-COR	0.106	0.0138	0.034	0.1448	0.0004	0.0253	0.1345	0.0212	0.0266	0.1052	0.1031	0.0886	0.1965
	MIN-CON	0.039	0.018	0.0478	0.1046	0.0286	0.0767	0.1148	0.0598	0.0597	0.1249	0.1133	0.0796	0.1331
COVID-19 period (Apr 1, 2020 to Feb 23, 2022)	MIN-VAR	0.0874	0.0066	0.1834	0.2568	0	0	0.0126	0.1768	0.0005	0.041	0.0227	0.064	0.1483
	MIN-COR	0.0728	0	0.0437	0.1305	0.0223	0.044	0.1744	0.0797	0.0196	0.0918	0.1053	0.0708	0.1449
	MIN-CON	0.0728	0	0.0428	0.1046	0.0388	0.0683	0.1418	0.0671	0.043	0.0982	0.1028	0.0853	0.1346
Russia-Ukraine war period	MIN-VAR	0.1168	0.0482	0.1972	0.2104	0	0	0.0101	0.0651	0	0.0697	0.0187	0	0.2638
	MIN-COR	0.1416	0.0766	0.0809	0.1038	0	0.029	0.1588	0.0759	0.0043	0.0657	0.1372	0.0001	0.1262
	MIN-CON	0.0779	0.045	0.046	0.1275	0.0231	0.0599	0.1581	0.0909	0.0342	0.1027	0.0948	0.0375	0.1025

Figures 7.a, 7.b and 7.c plot the dynamic portfolio weights for investment strategy C under the three approaches – MIN-VAR, MIN-COR and MIN-CON, and Table 11 summarizes the average weights during stress and stress-free periods. The results show that the structures of the MIN-COR and MIN-CON portfolios are largely similar, while the structure of the MIN-VAR portfolio is quite different.

More interestingly, we find that the three approaches do not share the same allocation tactics. Indeed, while the MIN-VAR portfolio method favors regional indices and assigns them relatively more weight over all the periods examined, the MIN-COR and MIN-CON methods favor commodities instead. Specifically, the MIN-VAR method assigns 55% to regional indices and 45% to short-term commodity futures during the pre-COVID-19 period, 50% to regional indices and 50% to short-term commodity futures during the COVID-19 crisis period, 53% to regional indices and 47% to short-term commodity futures during the pre-war period, and 57% to regional indices and 43% to short-term commodity futures during the Russia-Ukraine war period. Regarding the MIN-COR portfolio method (respectively the MIN-CON method), it attributes 26% (respectively 25%) to regional indices and 74% (respectively 75%) to short-term commodity futures during the pre-COVID period, 30% (respectively 24%) to regional indices and 70% (respectively 76%) to short-term commodity futures during the COVID-19 period, 27% (respectively 26%) to regional indices and 73% (respectively 74%) to short-term commodity futures during the pre-war period, and 40% (respectively 32%) to regional indices and 60% (respectively 68%) to short-term commodity futures during the Russia-Ukraine war period.

We also find that the structure of the portfolio under the three approaches (i) is not stable over time, and (ii) has experienced two major changes: it shifted once following the COVID-19 crisis and a second time following the onset of the Russian-Ukrainian war. The portfolio structure suggested by the MIN-VAR method during the pre-COVID-19 crisis period assigns relatively more weight to **GLD1** (about 27.25% on average), **APC** (about 22.46%), **GCC** (about 18.62%) and **NAM** (about 10.56%), while the weight of **LAM** and **WTI** are null or almost null. Over the same pre-COVID-19 stress-free period, the MIN-COR and MIN-CON methods suggest (on average) weights of 17.57% and 13.55% respectively for **GAS1**, 15.21% and 11.55% respectively for **GLD1**, and 12.33% and 11.54% respectively for **WHT1**. Once again, this result is quite expected as the natural gas market is the most disconnected and decoupled from other commodity markets.

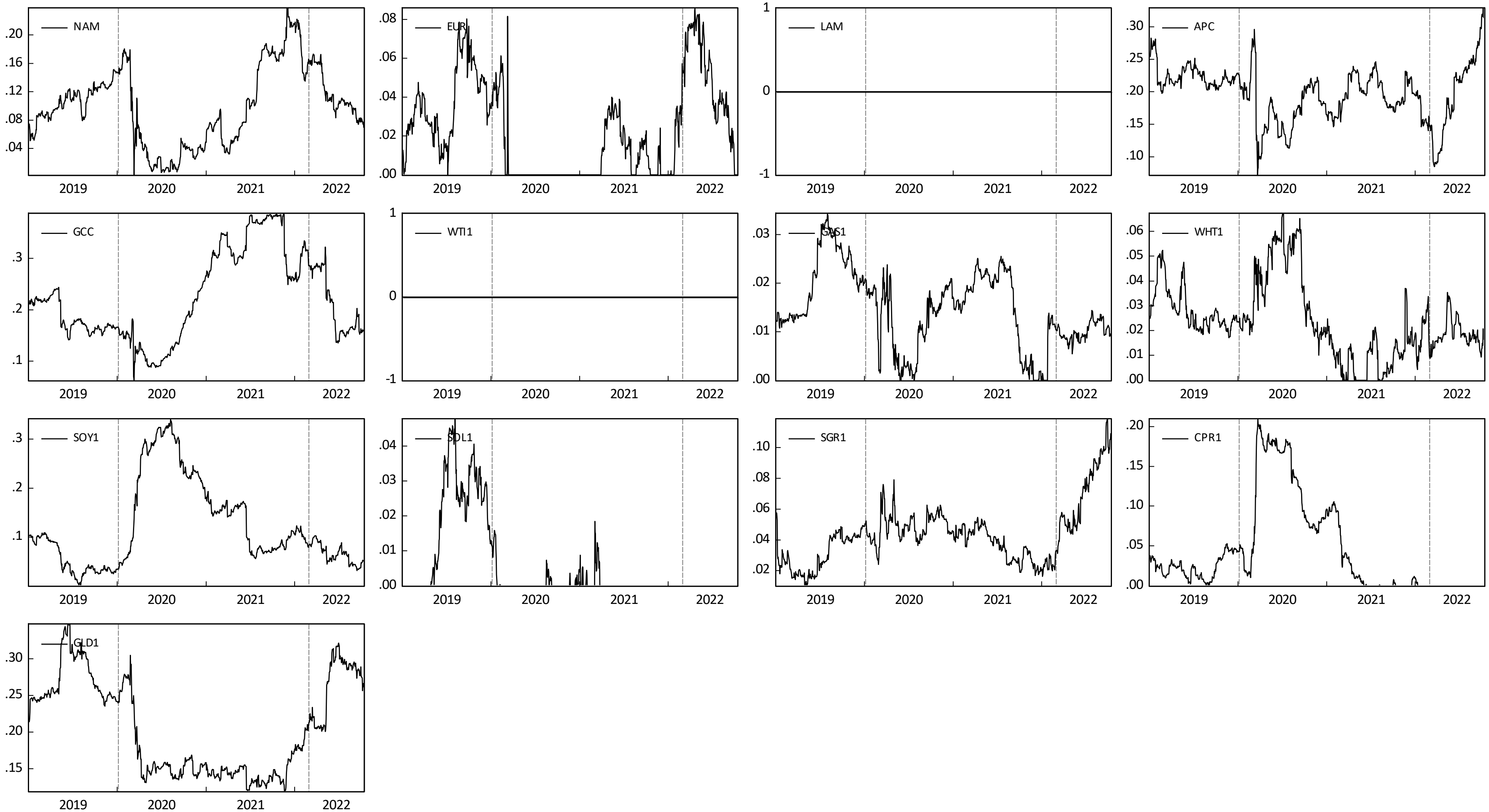
[**Insert Figures 7.a, 7.b and 7.c about here**]

Figures 7.a, 7.b and 7.c shows that the dynamic portfolio weights have shifted significantly during the COVID-19 crisis period. During the first quarter of 2020, the MIN-VAR portfolio suggests a significant increase (respectively a decrease) in the weights of **WHT1**, **SOY1**, and **CPR1** (respectively in the weights of **NAM**, **EUR**, **APC** and **GLD1**). As for **LAM**, **WTI1** and **SOL1**, their weight in the MIN-VAR portfolio is almost 0% throughout the COVID-19 period. Over the same COVID-19 crisis period, the MIN-COR portfolio assigns the highest weights to **GLD1** (19.65% on average), **GCC** (14.48% on average) and **GAS1** (13.45% on average), and the lowest weights to **LAM** (almost 0%) and **EUR** (1% on average). As for MIN-CON portfolio approach, it assigns the highest weights to **GLD1** (13.31% on average), **SGR1** (12.49% on average), **GAS1** (11.48% on average) and **WHT1** (11.33% on average), and the lowest weights to **EUR** (1.8% on average) and **LAM** (2.86% on average). Moreover, the weight of **EUR** dropped to almost 0% under three approaches during the COVID-19 crisis period. Indeed, a shock in one market can result in a change in investors' perceptions of the vulnerability and resilience of other markets. Indeed, during the first days of the COVID-19 pandemic, market participants did not give too much importance to this a new strain of coronavirus and its potential consequences. Initially the global perception was that this virus would be contained within China only. For this reason, the risk of leakage beyond China was not taken seriously enough (Ozili and Arun, 2020). However, attention paid to this new disease increased considerably after January 20th, 2020, when the Chinese health authorities warned that the virus can be transmitted human-to-human, with each patient infecting two or three others on average. Thus, the virus got out of China and hit the entire planet through people movement and social interactions. The facts that the COVID-19 is highly infectious, and several European countries have become infected areas, shifted the attention of market participants. The speed and extent to which the COVID-19 has spread across Europe seems to have damaged the market sentiment about the resilience of the global economy, triggering the spread of the “bad news” to all stock markets around the world by a domino effect. This may explain, at least in part, the drop in the weight of EUR during the first quarter of 2020.

[**Insert Figure 8 about here**]

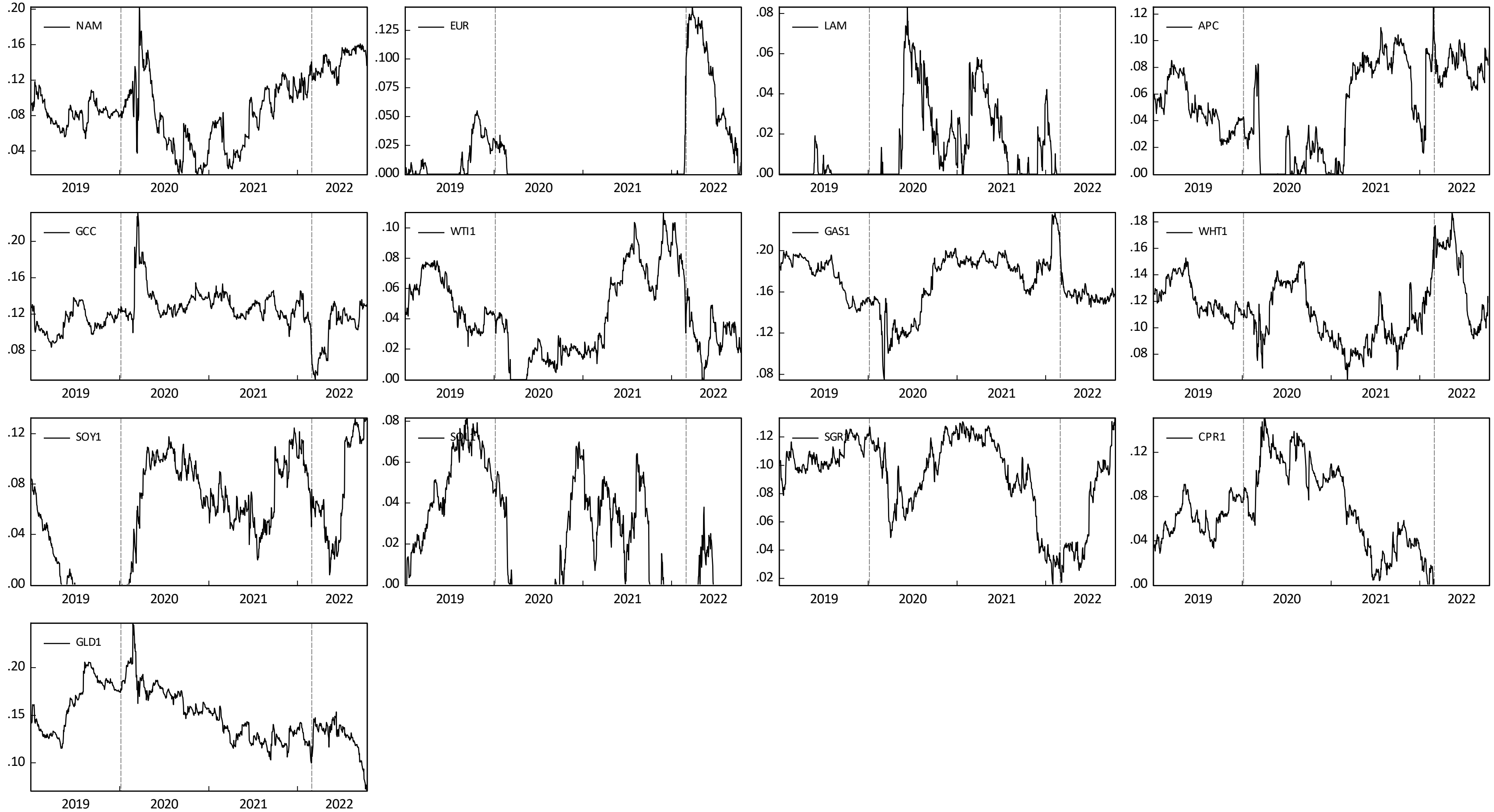
Figure 6 plots the performance, in terms of cumulative return, of the three alternative portfolio structures for Strategy C's panel of assets. The plot shows that the three portfolio methods display largely equivalent performances until early 2020, which corresponds to a stress-free period. Furthermore, all three approaches share the same underlying dynamics, including a significant decline in portfolio performance in the first quarter of 2020, followed by a pattern of sustained growth through February 2022, before dropping in the wake of the onset of the war in Ukraine. The MIN-VAR method provides investors with the relatively highest performance during the COVID-19 crisis period, and with the lowest performance during the Russia-Ukraine war period. The MIN-COR portfolio method provides investors with the lowest performance during all the pre-war subperiods. As for the MIN-CON portfolio method, it provides investors with the highest performance during the Russia-Ukraine war period.

Fig. 7.a. Strategy C - Weights [Minimum variance portfolio]



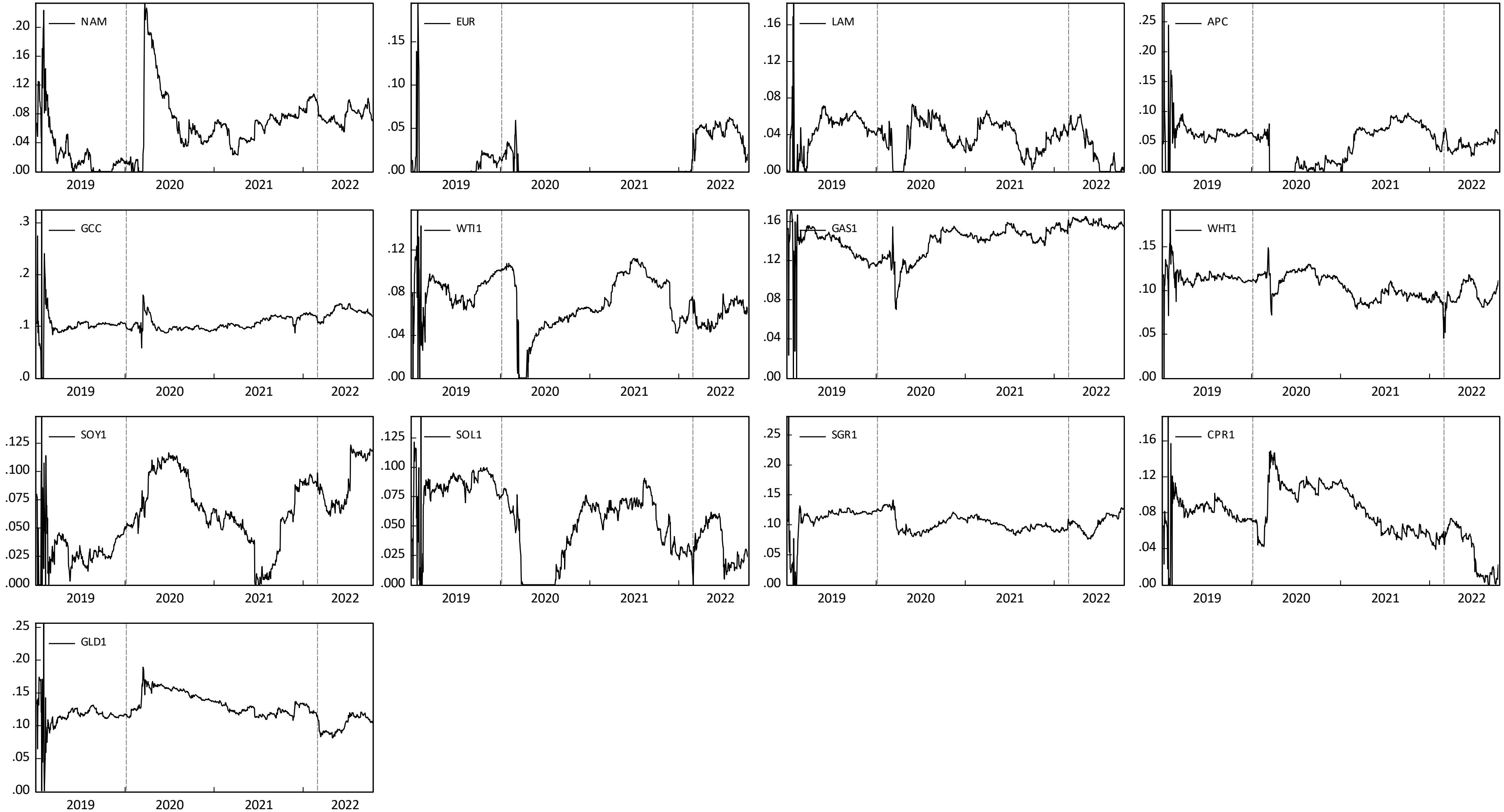
Notes: Strategy C consists of investing in regional stock indices and short-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI1 stands for front NYMEX WTI crude oil futures contract. GAS1 stands for front NYMEX Natural Gas futures contract. WHT1 stands for front CBOT wheat futures contract. SOY1 stands for front CBOT soybeans futures contract. SOL1 stands for front CBOT soybean futures contract. SGR1 stands for front ICE sugar #11 futures contract. CPR1 stands for front COMEX copper futures contract. GLD1 stands for front COMEX gold futures contract. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 7.b. Strategy C - Weights [Minimum correlation portfolio]



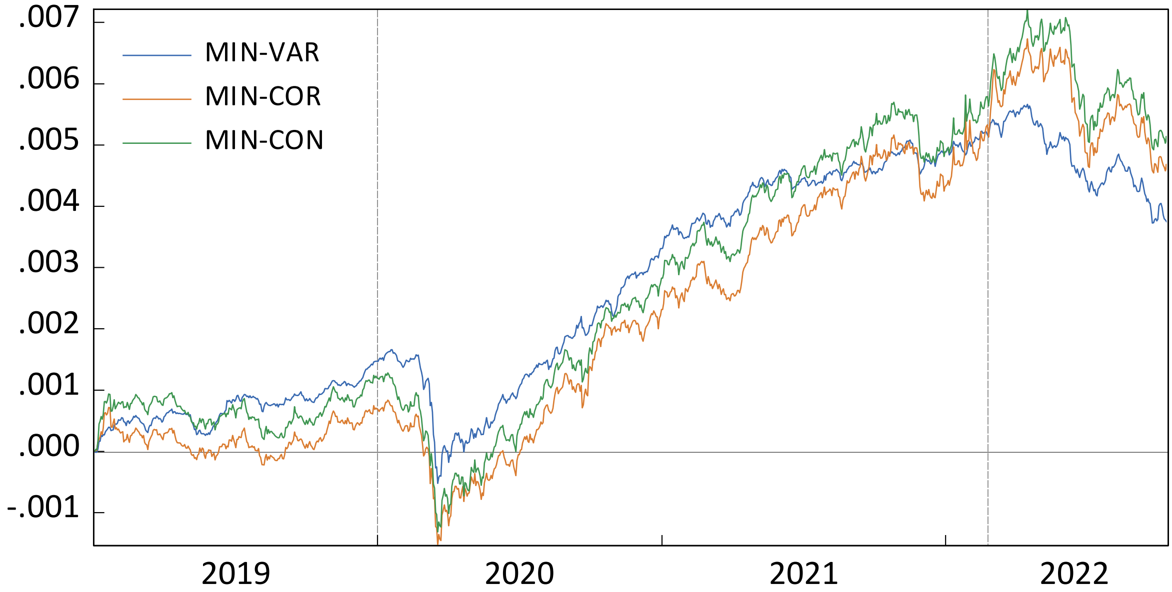
Notes: Strategy C consists of investing in regional stock indices and short-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI1 stands for front NYMEX WTI crude oil futures contract. GAS1 stands for front NYMEX Natural Gas futures contract. WHT1 stands for front CBOT wheat futures contract. SOY1 stands for front CBOT soybeans futures contract. SOL1 stands for front CBOT soybean futures contract. SGR1 stands for front ICE sugar #11 futures contract. CPR1 stands for front COMEX copper futures contract. GLD1 stands for front COMEX gold futures contract. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 7.c. Strategy C - Weights [Minimum connectedness portfolio]



Notes: Strategy C consists of investing in regional stock indices and short-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI1 stands for front NYMEX WTI crude oil futures contract. GAS1 stands for front NYMEX Natural Gas futures contract. WHT1 stands for front CBOT wheat futures contract. SOY1 stands for front CBOT soybeans futures contract. SOL1 stands for front CBOT soybean futures contract. SGR1 stands for front ICE sugar #11 futures contract. CPR1 stands for front COMEX copper futures contract. GLD1 stands for front COMEX gold futures contract. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 8 Strategy C - Cumulative returns



Notes: MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

- *Strategy D*

We now analyse diversification strategy D, which is composed of regional indices (**NAM**, **EUR**, **LAM**, **APC**, **GCC**) and long-term commodity futures contracts (**WTI70**, **GAS70**, **WHT12**, **SOY17**, **SOL20**, **SGR8**, **CPR10**, and **GLD12**).

Table 12: Average portfolio weights during stress and stress-free periods - Strategy D

Period	Method	NAM	EUR	APC	GCC	LAM	WTI70	GAS70	SOY17	SOL20	SGR8	WHT12	CPR10	GLD12
Pre-COVID-19 stress-free period	MIN-VAR	0.0681	0.0001	0.083	0.1191	0	0.0511	0.1153	0.1267	0.0307	0.0637	0.0843	0.0309	0.227
	MIN-COR	0.0053	0.0641	0.0856	0.0729	0.1144	0.0764	0.1418	0.0027	0.0801	0.0749	0.1222	0.0367	0.1229
	MIN-CON	0.0667	0.0009	0.0368	0.1058	0.0279	0.092	0.1268	0.0614	0.0732	0.1042	0.1103	0.0824	0.1117
COVID-19 period (Q1 2020)	MIN-VAR	0.0746	0.0004	0.0801	0.061	0	0.0213	0.0969	0.2396	0.0065	0.0759	0.1013	0.0423	0.2001
	MIN-COR	0.0517	0.0168	0.0444	0.0901	0.0341	0.0585	0.1538	0.0298	0.0522	0.1035	0.1243	0.0795	0.1613
	MIN-CON	0.0387	0.0289	0.0393	0.0929	0.0287	0.097	0.1177	0.0692	0.0801	0.1122	0.1089	0.0822	0.1043
COVID-19 period (Apr 1, 2020 to Feb 23, 2022)	MIN-VAR	0.0605	0.002	0.0958	0.1297	0	0.0388	0.1002	0.233	0.019	0.0804	0.0761	0.0381	0.1264
	MIN-COR	0.0648	0.0022	0.036	0.0935	0.0037	0.0752	0.1962	0.0929	0.0335	0.0851	0.0995	0.0777	0.1397
	MIN-CON	0.076	0.0056	0.0415	0.1042	0.0095	0.0865	0.1281	0.0798	0.0603	0.1003	0.1126	0.077	0.1186
Russia-Ukraine war period	MIN-VAR	0.1033	0.0056	0.1327	0.131	0	0.0377	0.0482	0.1019	0	0.1895	0.0264	0	0.2236
	MIN-COR	0.1459	0.0568	0.0489	0.0922	0	0.0781	0.1413	0.0198	0.0438	0.0986	0.1597	0.0143	0.1005
	MIN-CON	0.0652	0.0329	0.0554	0.1169	0.0102	0.0999	0.1346	0.0415	0.0602	0.1076	0.1157	0.0348	0.1251

Figures 9.a, 9.b and 9.c plot the dynamic portfolio weights for investment strategy D under the three approaches – MIN-VAR, MIN-COR and MIN-CON, and Table 12 summarizes the average weights during stress and stress-free periods. Once again, the results show that the structures of the MIN-COR and MIN-CON portfolios are largely similar, while the structure of the MIN-VAR portfolio is quite different. More interestingly, and unlike strategy C, we find that the three approaches share broadly the same allocation tactics. Indeed, the three portfolio methods favor long-term commodity futures and assigns them relatively more weight over all the periods examined. Specifically, the MIN-VAR method assigns 27% to

regional indices and 73% to long-term commodity futures during the pre-COVID-19 period, 22% to regional indices and 58% to long-term commodity futures during the COVID-19 crisis period, 29% to regional indices and 71% to long-term commodity futures during the pre-war period, and 37% to regional indices and 63% to long-term commodity futures during the Russia-Ukraine war period. Regarding the MIN-COR portfolio method, it attributes 34% to regional indices and 66% to commodity futures during the pre-COVID period, 24% to regional indices and 76% to commodities during the COVID-19 period, 20% to regional indices and 80% to commodities during the pre-war period, and 34% to regional indices and 66% to commodities during the Russia-Ukraine war period. As for the MIN-CON portfolio method, it attributes 24% to regional indices and 76% to commodity futures during the pre-COVID period, 23% to regional indices and 77% to commodities during the COVID-19 period, 24% to regional indices and 76% to commodities during the pre-war period, and 28% to regional indices and 72% to commodities during the Russia-Ukraine war period.

We also find that the estimated weights (i) are time-varying, and (ii) has experienced two major structural shifts: once following the COVID-19 crisis and a second time following the onset of the Russian-Ukrainian war. The portfolio structure suggested by the MIN-VAR method during the pre-COVID-19 crisis period assigns relatively more weight to **GLD12** (about 22.7% on average), **SOY17** (about 12.67%), **GCC** (about 11.91%) and **GAS70** (about 11.53%), while the weight of **LAM** and **EUR** are null or almost null. Over the same pre-COVID-19 stress-free period, the MIN-COR and MIN-CON methods suggest (on average) weights of 14.18% and 12.68% respectively for **GAS70**, 12.29% and 11.17% respectively for **GLD12**, and 12.22% and 11.03% respectively for **WHT12**.

[**Insert Figures 9.a, 9.b and 9.c about here**]

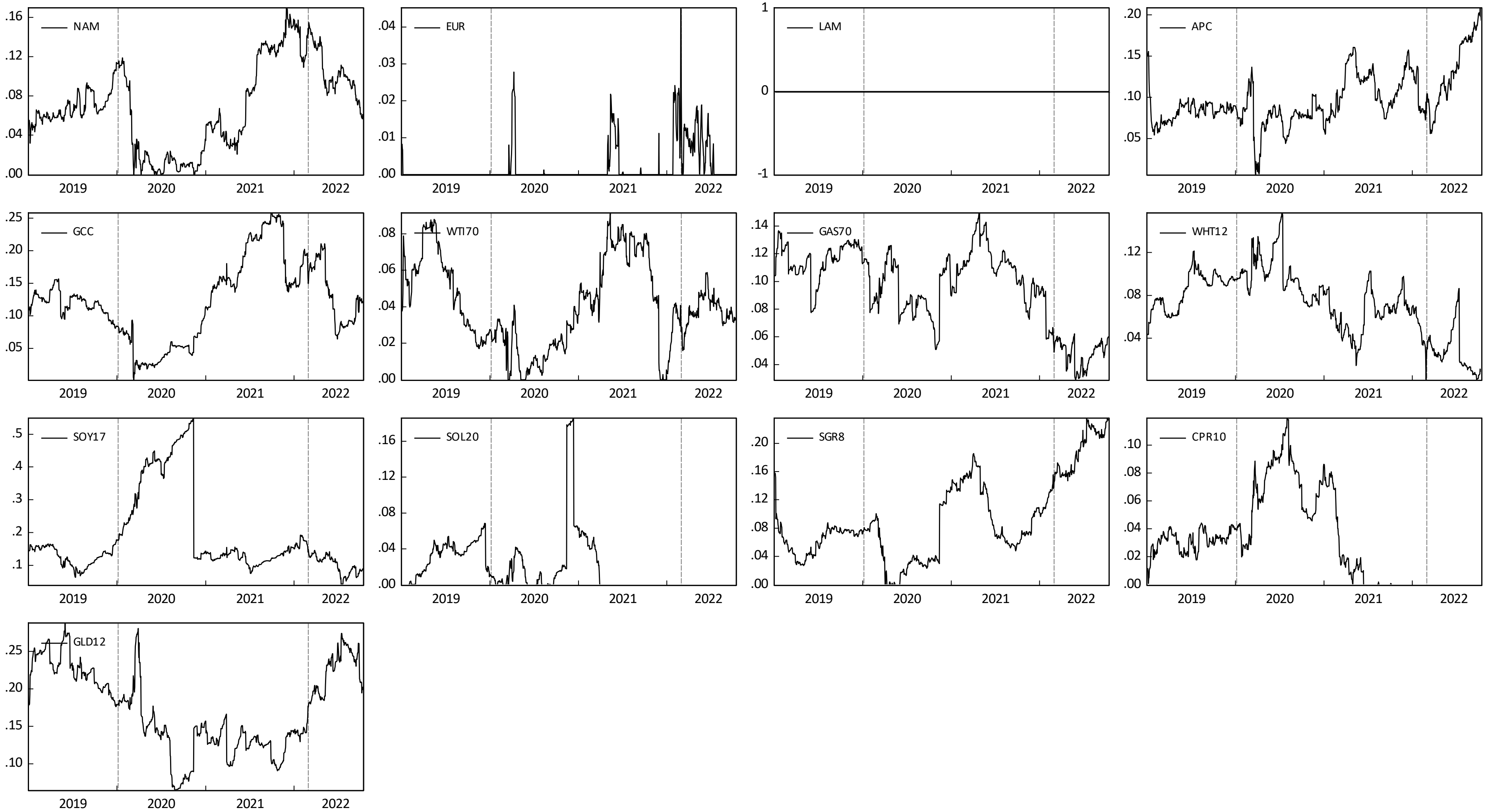
Figures 9.a, 9.b and 9.c show that the dynamic portfolio weights have shifted significantly during the COVID-19 crisis period. Over the COVID-19 period, the MIN-VAR portfolio method assigns the highest weights to **SOY17** (23.96% on average), **GLD12** (20.01%) and **WHT12** (10.13%), and the lowest weights to **LAM** (0% throughout the COVID-19 period), **EUR** (almost 0%) and **SOL20** (0.65%). Over the same period, the MIN-COR method assigns the highest weights to **GLD12** (16.13% on average), **GAS70** (15.38%), **WHT12** (12.43%) and **SGR8** (10.35%), and the lowest weight to **EUR** (1.6%). While the MIN-CON portfolio method suggests a significant temporary increase (respectively a decrease) in the weights of **NAM**, **EUR** and **CPR10** (respectively in the weights of **LAM**, **APC**, **WTI70** and **WHT12**) during the first quarter of 2020, it largely share the same structure as the MIN-COR method. Indeed, the MIN-CON method assigns the highest weights to **GAS70** (11.77% on average), **SGR8** (11.22%), **WHT12** (10.89%) and **GLD12** (10.43%), and the lowest weight to **LAM** and **EUR** (2.8%).

During the ongoing Russia-Ukraine war period, although the MIN-COR and MIN-CON methods suggest fairly similar structures, we nevertheless notice that the MIN-COR method assigns a relatively large weight to **NAM** (14.59% on average during the war period), while the MIN-CON method assigns a relatively small weight to this asset (6.5% on average). On the other hand, the MIN-VAR portfolio approach assigns the highest weights to **GLD12** (22.36% on average), **SGR8** (18.95%), **APC** (13.27%) and **GCC** (13.1%), and the lowest weights to **LAM**, **SOL20** and **CPR10** (0% throughout the war period). A visual inspection of the weights plots reveals that the outbreak of the Russia-Ukraine war was a major shock that caused portfolio managers to abruptly change the structure of their portfolios. Indeed, we note that, following the war in Ukraine, the three portfolio methods suggest an increase in the weight of **EUR** instantaneously with the onset of the war. This result potentially reflects the fact that, during this period of high geopolitical tensions, market participants had considered the European market as a safe haven.

[**Insert Figure 10 about here**]

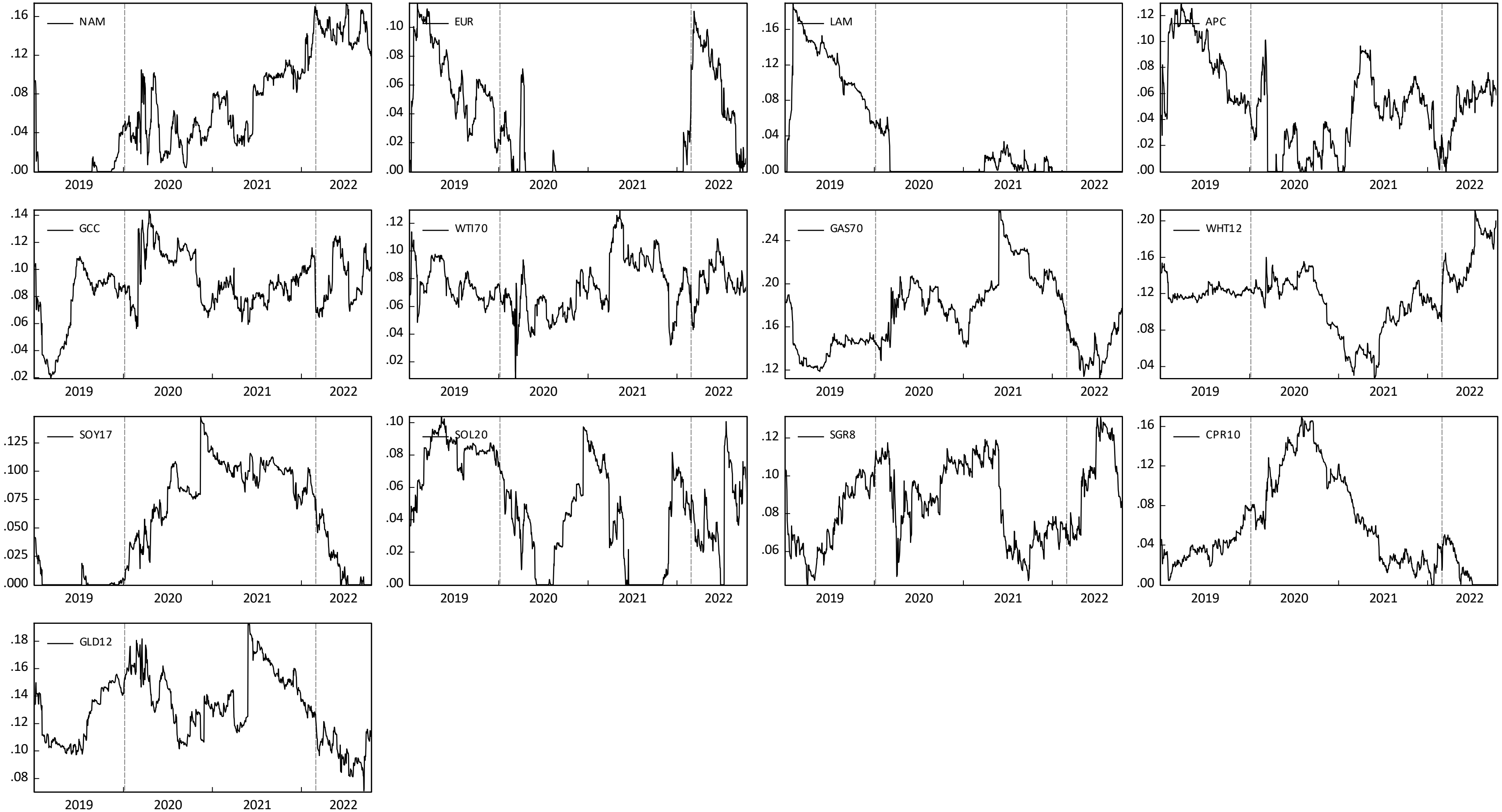
Figure 10 plots the performance, in terms of cumulative return, of the three alternative portfolio structures for Strategy D's panel of assets. The plot shows that all three approaches share the same underlying dynamics, including a significant decline in portfolio performance in the first quarter of 2020, followed by a pattern of sustained growth through February 2022, before dropping in the wake of the onset of the war in Ukraine. The MIN-CON portfolio method provides investors with the relatively highest performance over the sample period. The MIN-COR portfolio method provides investors with the lowest performance during all the pre-war subperiods. As for the MIN-VAR portfolio method, it provides investors with the lowest performance during the Russia-Ukraine war period.

Fig. 9.a. Strategy D - Weights [Minimum variance portfolio]



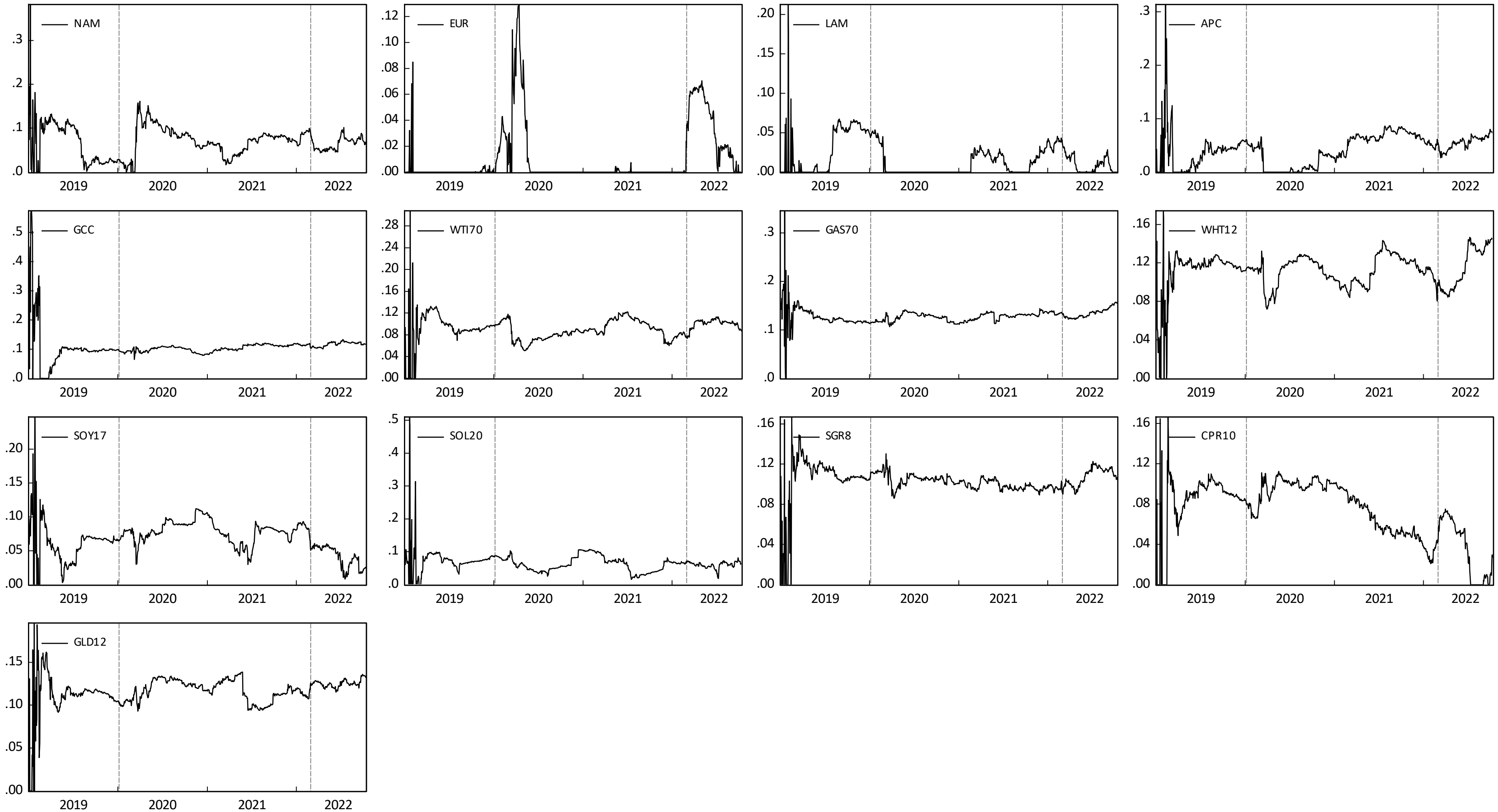
Notes: Strategy D consists of investing in regional stock indices and long-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI70 stands for NYMEX WTI crude oil futures with maturity 70 months. GAS70 stands for NYMEX Natural Gas futures with maturity 70 months. WHT12 stands for CBOT wheat futures with maturity 29 months. SOY17 stands for CBOT soybeans futures with maturity 29 months. SOL20 stands for CBOT soybean futures with maturity 31 months. SGR8 stands for ICE sugar #11 futures with maturity 22 months. CPR10 stands for COMEX copper futures with maturity 10 months. GLD12 stands for COMEX gold futures with maturity 24 months. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 9.b. Strategy D - Weights [Minimum correlation portfolio]



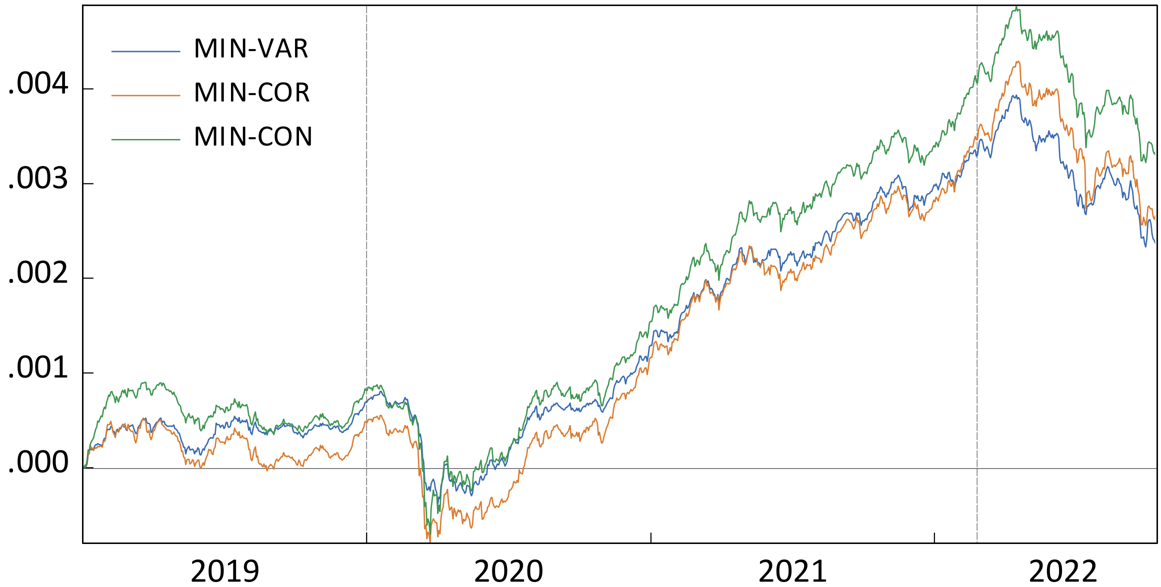
Notes: Strategy D consists of investing in regional stock indices and long-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI70 stands for NYMEX WTI crude oil futures with maturity 70 months. GAS70 stands for NYMEX Natural Gas futures with maturity 70 months. WHT12 stands for CBOT wheat futures with maturity 29 months. SOY17 stands for CBOT soybeans futures with maturity 29 months. SOL20 stands for CBOT soybean futures with maturity 31 months. SGR8 stands for ICE sugar #11 futures with maturity 22 months. CPR10 stands for COMEX copper futures with maturity 10 months. GLD12 stands for COMEX gold futures with maturity 24 months. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 9.c. Strategy D - Weights [Minimum connectedness portfolio]



Notes: Strategy D consists of investing in regional stock indices and long-run commodity futures. NAM, EUR, LAM, APC, and GCC stand for MSCI North America Index, MSCI Europe Index, MSCI Latin America Index, MSCI Asia Pacific Index, MSCI GCC Countries Combined Index, respectively. WTI70 stands for NYMEX WTI crude oil futures with maturity 70 months. GAS70 stands for NYMEX Natural Gas futures with maturity 70 months. WHT12 stands for CBOT wheat futures with maturity 29 months. SOY17 stands for CBOT soybeans futures with maturity 29 months. SOL20 stands for CBOT soybean futures with maturity 31 months. SGR8 stands for ICE sugar #11 futures with maturity 22 months. CPR10 stands for COMEX copper futures with maturity 10 months. GLD12 stands for COMEX gold futures with maturity 24 months. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 10 Strategy D - Cumulative returns



Notes: MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

- *Strategy E*

We now analyse diversification strategy E, which is composed of a representative index of the global stock market (**WRD**), short-term commodity futures (**WTI1**, **GAS1**, **WHT1**, **SOY1**, **SOL1**, **SGR1**, **CPR1**, and **GLD1**) and long-term commodity futures contracts (**WTI70**, **GAS70**, **WHT12**, **SOY17**, **SOL20**, **SGR8**, **CPR10**, and **GLD12**).

Table 13: Average portfolio weights during stress and stress-free periods - Strategy E

Period	Method	WRD	WTI1	WTI70	GAS1	GAS70	SOY1	SOY17	SOL1	SOL20	SGR1	SGR8	WHT1	WHT12	CPR1	CPR10	GLD1	GLD12
Pre-COVID-19 stress-free period	MIN-VAR	0.118	0.000	0.056	0.013	0.088	0.000	0.157	0.000	0.044	0.001	0.056	0.000	0.095	0.000	0.187	0.034	0.152
	MIN-COR	0.077	0.006	0.069	0.138	0.123	0.015	0.001	0.001	0.054	0.066	0.055	0.034	0.100	0.055	0.064	0.070	0.074
	MIN-CON	0.054	0.040	0.065	0.096	0.108	0.017	0.034	0.031	0.062	0.062	0.051	0.059	0.078	0.008	0.120	0.055	0.062
COVID-19 period (Q1 2020)	MIN-VAR	0.083	0.000	0.031	0.006	0.066	0.001	0.217	0.000	0.023	0.003	0.053	0.000	0.094	0.000	0.277	0.040	0.106
	MIN-COR	0.064	0.001	0.046	0.100	0.128	0.006	0.018	0.002	0.052	0.075	0.035	0.051	0.082	0.003	0.172	0.081	0.085
	MIN-CON	0.036	0.031	0.084	0.099	0.113	0.015	0.048	0.014	0.074	0.069	0.061	0.063	0.070	0.044	0.053	0.053	0.073
COVID-19 period (Apr 1, 2020 to Feb 23. 2022)	MIN-VAR	0.085	0.001	0.059	0.005	0.097	0.025	0.198	0.000	0.029	0.002	0.060	0.000	0.114	0.004	0.181	0.008	0.133
	MIN-COR	0.034	0.059	0.023	0.104	0.158	0.030	0.071	0.006	0.021	0.031	0.039	0.037	0.054	0.070	0.141	0.010	0.114
	MIN-CON	0.047	0.048	0.050	0.104	0.110	0.030	0.059	0.023	0.048	0.046	0.061	0.035	0.069	0.129	0.013	0.059	0.069
Russia-Ukraine war period	MIN-VAR	0.161	0.000	0.059	0.003	0.057	0.004	0.103	0.000	0.004	0.004	0.206	0.000	0.057	0.007	0.061	0.111	0.163
	MIN-COR	0.164	0.010	0.037	0.113	0.115	0.042	0.006	0.007	0.043	0.022	0.073	0.042	0.106	0.104	0.000	0.062	0.055
	MIN-CON	0.095	0.038	0.060	0.102	0.105	0.038	0.020	0.026	0.042	0.027	0.075	0.041	0.057	0.159	0.000	0.026	0.089

Figures 11.a, 11.b and 11.c plot the dynamic portfolio weights for investment strategy E under the three approaches, and Table 13 summarizes the average weights during stress and stress-free periods. The results show that the three portfolio methods are quite similar and largely share the same allocation structures. Indeed, all three approaches assign relatively higher weights to long-term commodity futures contracts. Specifically, the MIN-VAR method assigns 11.8% to the global stock index, 4.8% to short-term commodity futures and 83.4% to long-term commodity futures during the pre-COVID-19 period, 8.3% to the global stock index, 5% to short-term commodity futures and 86.7% to long-term commodity futures during the COVID-19 crisis period, 8.5% to the global stock index, 4.5% to short-term commodity futures and 87% to long-term commodity futures during the pre-war period, and 16.1% to the global stock index, 12.9% to short-term commodity futures and 71% to long-term commodity futures during the Russia-Ukraine war period. Regarding the MIN-COR method, it assigns 7.5% to the global stock index, 38.5% to short-term commodity futures and 54% to long-term

commodity futures during the pre-COVID-19 period, 6.4% to the global stock index, 31.8% to short-term commodity futures and 61.8% to long-term commodity futures during the COVID-19 crisis period, 3.4% to the global stock index, 34.5% to short-term commodity futures and 62.1% to long-term commodity futures during the pre-war period, and 16.4% to the global stock index, 40.1% to short-term commodity futures and 43.5% to long-term commodity futures during the Russia-Ukraine war period. Finally, the MIN-CON method assigns 5.4% to the global stock index, 36.6% to short-term commodity futures and 58% to long-term commodity futures during the pre-COVID-19 period, 3.6% to the global stock index, 38.8% to short-term commodity futures and 57.6% to long-term commodity futures during the COVID-19 crisis period, 4.7% to the global stock index, 47.4% to short-term commodity futures and 47.9% to long-term commodity futures during the pre-war period, and 9.5% to the global stock index, 45.7% to short-term commodity futures and 44.8% to long-term commodity futures during the Russia-Ukraine war period.

[**Insert Figures 11.a, 11.b and 11.c about here**]

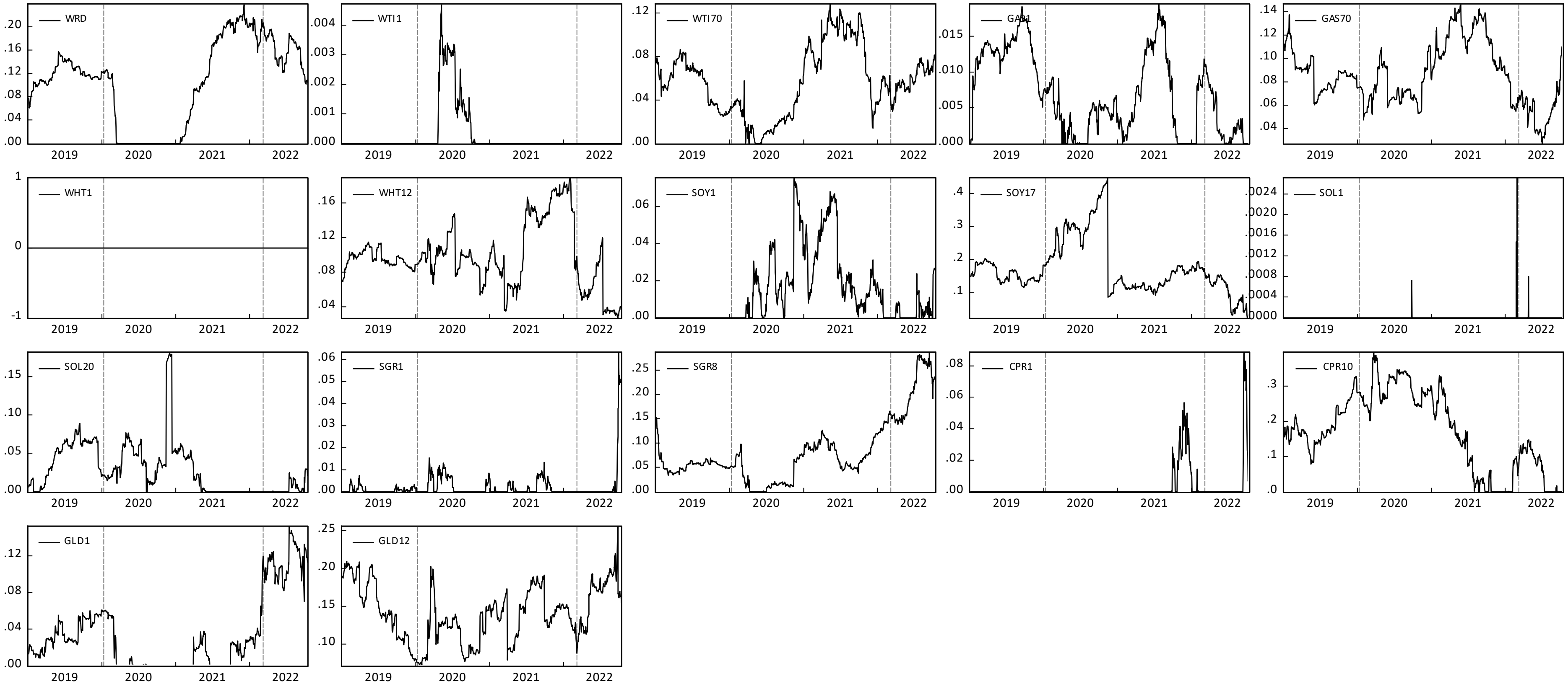
Moreover, the results show that the weights structures of the portfolio under the three approaches have experienced two major shifts: the first following the COVID-19 crisis and the second following the onset of the Russian-Ukrainian conflict. More specifically, all three approaches suggest that the weight of **WRD** fell during the period of COVID-19 (especially during the first quarter of 2020) and jumped with the outbreak of the war. Indeed, during the very first phase of the COVID-19 pandemic, market participants did not attach much importance to the possible consequences of this new strain of coronavirus and its potential consequences. It is worth remembering that among the ten most likely risks mentioned in the World Economic Forum's *Global Risks Report 2020*, released in January 2020, the risk of "infectious diseases" was ranked tenth and considered fairly unlikely. Initially the global perception was that this virus would be contained within China only. For this reason, the risk of leakage beyond China was not taken seriously enough (Ozili and Arun, 2020). But two months after the report was published, the COVID-19 pandemic was present in most countries of the world, abruptly changing the outlook unexpectedly and portending heavy human, economic and financial consequences (Elliot, 2020). However, when the Chinese health authorities warned, on January 20th, 2020, that this new strain of coronavirus can be transmitted human-to-human, with each patient infecting two or three others on average, attention paid to this new disease increased considerably. The speed and extent to which the COVID-19 virus has spread across the world have damaged the market sentiment about the resilience of the global economy, triggering the spread of the "bad news" to all stock markets around the world by a domino effect. This may explain, at least in part, the drop in the weight of **WRD** during the first quarter of 2020. The weight structure has shifted again following the onset of the Russian-Ukranian war and the subsequent broad economic and financial sanctions decided by several countries against Russia.[6] Indeed, we find that, according to all three portfolio approaches, the weight of **WRD** jumped with the onset of the war.

[**Insert Figure 12 about here**]

[6] For more details on government sanctions and measures taken by major corporations and organisations around the world against Russia after its invasion of Ukraine, we refer the interested readers to https://graphics.reuters.com/UKRAINE-CRISIS/SANCTIONS/byvrjenzmve/

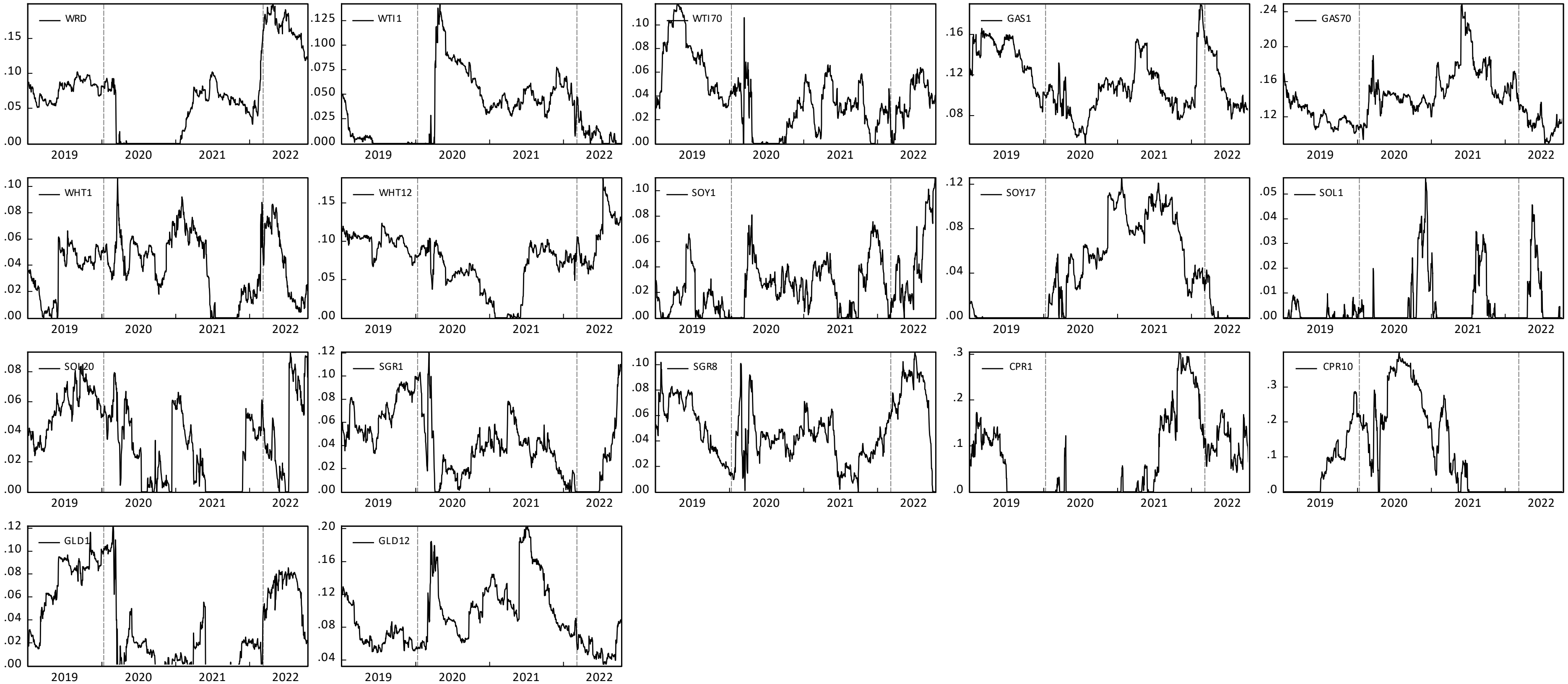
Figure 12 plots the performance, in terms of cumulative return, of the three alternative portfolio structures for Strategy E's panel of assets. The plot shows that the three portfolio methods display largely equivalent performances until early 2020, including a dip in the first quarter of 2020 followed by a pattern of sustained increase until the end of the sample period. Moreover, the MIN-CON portfolio slightly outperforming the MIN-VAR and the MIN-COR portfolios. Furthermore, all three approaches share the same underlying dynamics, including a significant decline in portfolio performance in the first quarter of 2020, followed by a pattern of sustained growth through February 2022, before dropping in the wake of the onset of the war in Ukraine. The MIN-COR and MIN-CON methods provide investors with the relatively highest performance during the COVID-19 crisis period, as well as the Russia-Ukraine conflict period, while the MIN-VAR portfolio method provides them with the lowest performance, especially during the war period.

Fig. 11.a. Strategy E - Weights [Minimum variance portfolio]



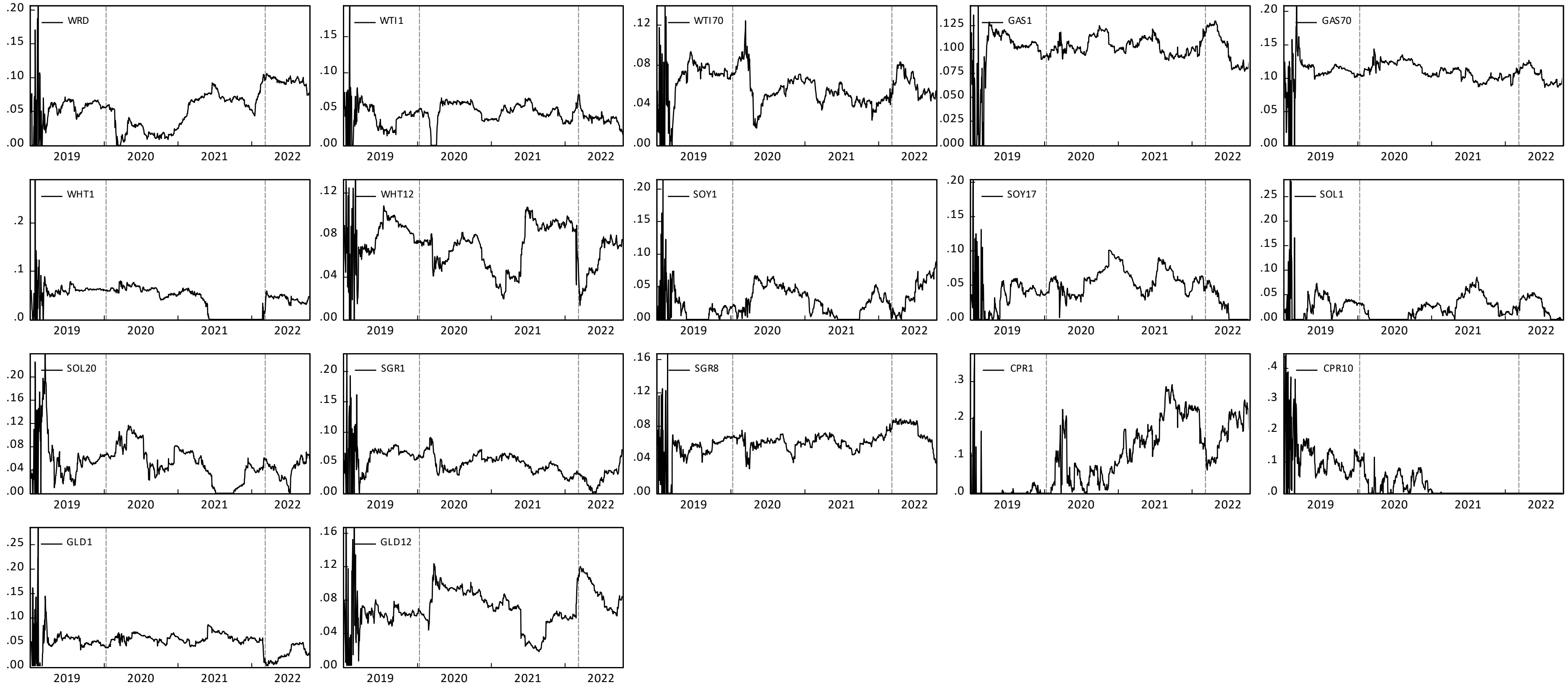
Notes: Strategy E consists of investing in a global stock index, as well as short-run and long-run commodity futures. WRD stands for MSCI ACWI Index. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS1 and GAS70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 11.b. Strategy E - Weights [Minimum correlation portfolio]



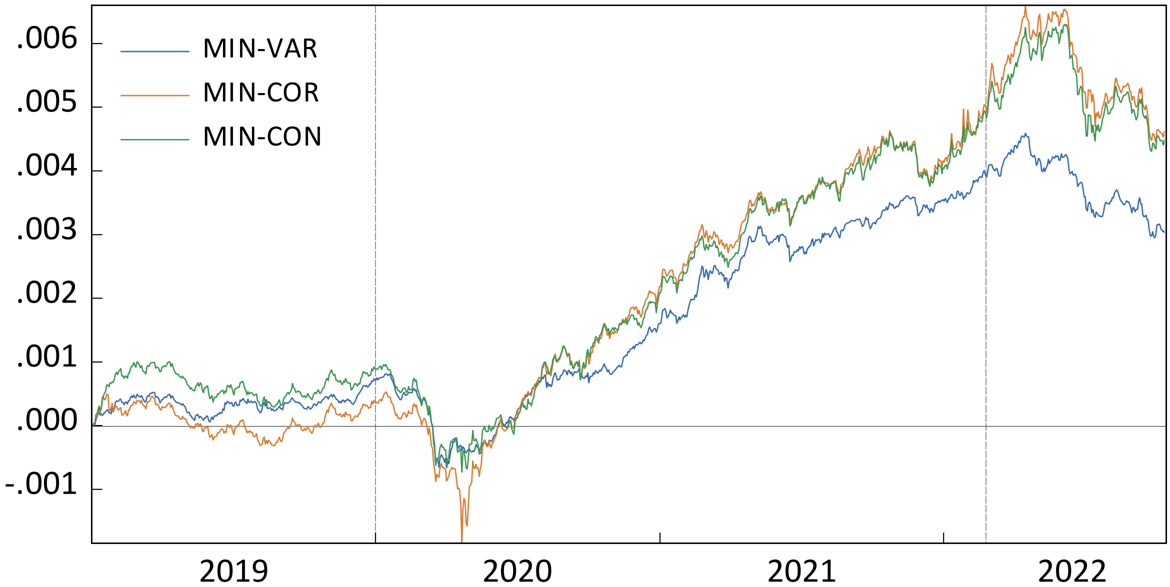
Notes: Strategy E consists of investing in a global stock index, as well as short-run and long-run commodity futures. WRD stands for MSCI ACWI Index. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS1 and GAS70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Fig. 11.c. Strategy E - Weights [Minimum connectedness portfolio]



Notes: Strategy E consists of investing in a global stock index, as well as short-run and long-run commodity futures. WRD stands for MSCI ACWI Index. WTI1 and WTI70 stand for NYMEX WTI crude oil futures with maturities 1 and 70 months, respectively. GAS1 and GAS70 stand for NYMEX Natural Gas futures with maturities 1 and 70 months, respectively. WHT1 and WHT12 stand for CBOT wheat futures with maturities 3 and 29 months, respectively. SOY1 and SOY17 stand for CBOT soybeans futures with maturities 1 and 29 months, respectively. SOL1 and SOL20 stand for CBOT soybean futures with maturities 1 and 31 months, respectively. SGR1 and SGR8 stand for ICE sugar #11 futures with maturities 3 and 22 months, respectively. CPR1 and CPR10 stand for COMEX copper futures with maturities 1 and 10 months. GLD1 and GLD12 stand for COMEX gold futures with maturities 2 and 24 months, respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

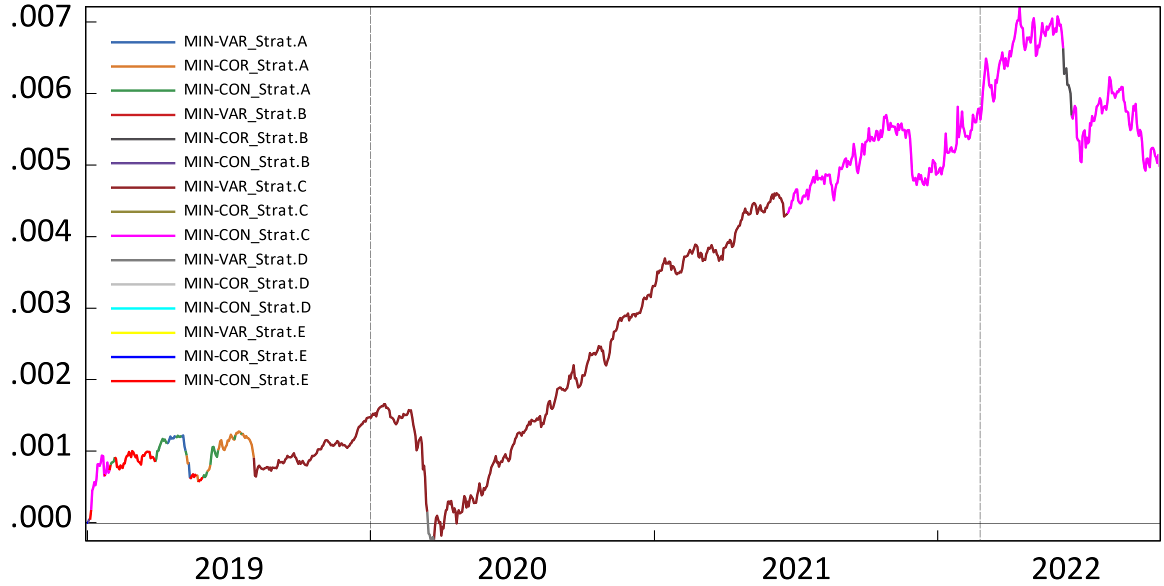
Fig. 12 Strategy E - Cumulative returns



Notes: MIN-VAR, MIN-COR, and MIN CON stand for minimum-variance portfolio (Markowitz, 1959), minimum-correlation portfolio (Christoffersen et al., 2014), and minimum-connectedness portfolio (Broadstock et al., 2022), respectively. The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

In the context of increasingly integrated financial markets, and the resulting systemic risk, it is certainly not trivial to question the most efficient portfolio allocation during stress and stress-free periods. Thus, by using a dataset covering a broad spectrum of global and regional stock indices and short- and long-term commodity futures contracts, this paper has studied five investment strategies using three portfolio methods (MIN-VAR, MIN-COR and MIN-CON), making fifteen potential allocations. Our results highlight the role of both the method and the underlying assets in diversification strategies. Figure 13 shows the best performing diversification strategies over the study period.

Fig. 13 Best performing diversification strategy over time



Notes: The first hatched vertical line (from left to right) corresponds to December 31, 2019, when the first case of COVID-19 was reported to the World Health Organization Country Office in China. The second hatched vertical line (from left to right) corresponds to 24 February 2022, when the war between Russia and Ukraine was initiated.

Our results show that during the pre-COVID stress-free period and through mid-2019, the different investment strategies were largely equivalent in terms of performance, and there is no dominant strategy. Furthermore, from mid-2019 to mid-2021, and going through the COVID-19 period, investment strategy C, which is composed of regional indices and short-term commodity futures contracts, under the MIN-VAR method, outperforms the other investment strategies. We further note that strategy C under the MIN-CON portfolio method outperforms all other investment strategies from mid-2021 (i.e., towards the end of the pre-war period) and during the Russian-Ukrainian war period.

4. Concluding remarks

This paper investigates the structure as well as the performance of different investment strategies using different portfolio methods, over the period January 1st, 2019-October 14th, 2022. To that end, we first used a TVP-VAR model to estimate the time-varying variance-covariance matrices, and the Generalized Forecast Error Variance Decomposition (GFEVD) framework to examine the connectedness structure among the considered assets. Then, we constructed different portfolios using three diversification strategies – the traditional minimum-variance portfolio strategy (Markowitz, 1952), the minimum-correlation portfolio strategy (Christoffersen, et al., 2014), and the minimum-connectedness portfolio strategy (Broadstock et al., 2022) – and compared them. In this study we assumed an investor interested in investing in (**i**) the global equity market, but also in (**ii**) regional equity markets (North America, Latin America, Europe, Asia-Pacific, GCC) and (**iii**) short- and long-term commodity futures markets (Oil, Natural Gas, Soybeans, Soybean Oil, Sugar, Wheat, Copper, Gold). Specifically, we assumed that the investor has a choice among five different investment strategies, and we examined the structure and performance of these strategies during recent stress and stress-free periods.

Regarding results, we provide a set of stylized facts on the heterogeneity of the impact of the different portfolio approaches used over the range of stress and stress-periods periods examined. These results can be summarized as follows. The connectedness network analysis reveals a relatively high level of integration between the global and regional stock markets, as well as between each of the short-term futures contracts and its long-term counterpart, supporting the hypothesis that stock and commodity markets are potentially weakly integrated. However, regional stock markets appear to be relatively much more integrated than commodity futures markets. Moreover, the connectedness analysis show that the natural gas futures market appears to be the most isolated market, i.e., it weakly affects (and is weakly affected by) the other markets. In addition, agricultural commodity futures markets appear to be broadly insensitive to shocks in non-agricultural commodity markets, supporting the thesis stating that commodity markets are potentially segmented. The connectedness analysis also reveals that the GCC stock market is largely disconnected from other regional stock markets, as it neither influences nor is influenced by other regional stock markets to any significant extent.

Regarding portfolio allocation strategies, the results suggest that there is no dominant strategy during the pre-COVID stress-free period and through mid-2019, i.e., that the performance of the different investment strategies were largely equivalent in terms of cumulative returns. However, from mid-2019 to mid-2021, and going through the COVID-19 period, the portfolio strategy composed of regional indices and short-term commodity futures contracts, under the Markowitz (1952) minimum-variance approach, outperforms the other investment strategies. We further find that the same portfolio strategy, but under the Broadstock et al. (2022) minimum-connectedness approach, outperforms all other investment strategies from mid-2021 and during the ongoing Russian-Ukrainian war period.

The main contributions of this study to the literature are twofold. First, our paper fills the gap in the literature by examining diversification strategies when considering both stock markets and commodities over different maturities, ranging from the short- to the long-term, as this may hide useful information in terms of fund allocation and risk management. Indeed, our results highlight the magnitude of linkages between stock and commodity futures markets, and provide markets participants with useful guidance in this regard. For example, as the natural gas market appears to be broadly isolated from other market, it may enable to use it to optimize portfolio diversification strategies. Second, the period under study includes recent episodes of high market uncertainty, including the COVID-19 period and the ongoing Russian-Ukrainian conflict, as well as periods of low market uncertainty. This allows us to track the dynamics of portfolio weights structure during stress as well as stress-free episodes. Our findings are of practical importance for market participants. Indeed, our results allow investors to better understand: (i) the sensitivity of the investment strategy's performance to the portfolio method used (MIN-VAR, MIN-COR or MIN-CON); and (ii) the extent to which the recent destabilizing episodes had influenced the structure of the portfolios, as well as their performance. In a context of increasing financial integration, a better understanding of potential diversification opportunities, as well as their implications in terms of underlying risk and return, in both stress and stress-free periods, is crucial for investors in order to achieve better and more efficient portfolio diversification.

References

Ankrim, E. M., & Hensel, C. R. (1993). Commodities in asset allocation: A real-asset alternative to real estate? *Financial Analysts Journal*, 49(3), 20–29.

Antonakakis, N., Chatziantoniou, I., Gabauer D., (2020). Refined Measures of Dynamic Connectedness based on Time-Varying Parameter Vector Autoregressions. *Journal of Risk and Financial Management*, 13(4).

Arouri, M.E.H., Jawadi, F., Nguyen D.K., (2009), International Stock Return Linkages: Evidence from Latin American Markets, *European Journal of Economics, Finance and Administrative Sciences*, 11, 162-168.

Arouri, M.E.H., Jouini, J., and Nguyen, D.K., (2011), Volatility Spillovers between Oil Prices and Stock Sector Returns: Implications for Portfolio Management, *Journal of International Money and Finance*, 30(7), 1387-1405.

Authers, J., & Meyer, G., (2015). Investment: Revaluing Commodities. Financial Times (30 December): http://www.ft.com/intl/cms/s/0/a6ff2818-094c-11e5-8534-00144feabdc0.html#slide0

Babalos, V., and Stavroyiannis, S., (2015). Herding, anti-herding behaviour in metal commodities futures: a novel portfolio-based approach. *Applied Economics*. 47(46), 4952-4966.

Baumeister, C., Durinck, E., and Peersman, G., (2008). Liquidity, inflation and asset prices in a time-varying framework for the euro area, *National Bank of Belgium Working Paper Research*, no.142.

Bélaïd, F., Ben Amar, A., Goutte, S., & Guesmi, K., (2021). Emerging and Advanced Economies Market Behavior During the COVID-19 Crisis Era. International Journal of Finance and Economics.

Belke, A., & Dubova, I., (2018). International spillovers in global asset markets. *Economic Systems*, 42(1), 3–17.

Belousova, J., & Dorfleitner, G., (2012). On the diversification benefits of commodities from the perspective of euro investors. *Journal of Banking and Finance*, 36(9), 2455-2474.

Ben Amar, A., Bélaïd, F., Ben Youssef, A., & Guesmi, K., (2021). Connectedness among regional financial markets in the context of the COVID-19. Applied Economics Letters, 28(20), 1789-1796.

Ben Amar, A., Goutte, S., Isleimeyyeh, M., & Benkraiem, R., (2022). Commodity markets dynamics: What do cross-commodities over different nearest-to-maturities tell us? *International Review of Financial Analysis*, 82.

Bhardwaji, G., Gorton, G.B., & Rouwenhorst, K.G., (2016). Investor interest and the returns to commodity investing. *Journal of Portfolio Management*, 42(3), 44-55.

Bijsterbosch, M., and Falagiarda, M., (2014). Credit supply dynamics and economic activity in euro area countries: A time-varying parameter VAR analysis, *ECB Working Paper Series*, no.1714.

Broadstock, D.C., Chatziantoniou, I., Gabauer, D., (2022). Minimum Connectedness Portfolios and the Market for Green Bonds: Advocating Socially Responsible Investment

(SRI) Activity. In: Floros, C., Chatziantoniou, I. (eds) Applications in Energy Finance. Palgrave Macmillan, Cham.

Buyuksahin, B., & Robe, M. A. (2014). Speculators, commodities and cross-market linkages. *Journal of International Money and Finance*, *42*, 38-70.

Canova, F., and Pérez Forero, F.J., (2015). Estimating overidentified, nonrecursive, time-varying coefficients structural vector autoregressions, *Quantitative Economics*, 6(2), 359-384.

Carriero, A., Clark, T.E., and Marcellino, M., (2015). Realtime nowcasting with a Bayesian mixed frequency model with stochastic volatility, *Series A Statistics in Society*, 178(4), 837-862.

Cheung, C.S., & Miu, P., (2010). Diversification benefits of commodity futures. *Journal of International Financial Markets, Institutions and Money*, 20(5), 451-474.

Christoffersen, P., Errunza, V., Jacobs, K., & Jin, X., (2014). Correlation dynamics and international diversification benefits. *International Journal of Forecasting*, 30(3), 807-824.

Cogley, T., and Sargent, T.J., (2001). Evolving Post-World War II U.S. inflation dynamics, *NBER Macroeconomics Annual*, 331-373.

D'Agostino, A., Gambetti, L., and Giannone, D., (2013). Macroeconomic forecasting and structural change, *ECB Working Paper*, no.1167.

Daskalaki, C., Kostakis, A., & Skiadopoulos, G., (2014). Are there common factors in individual commodity futures returns? *Journal of Banking and Finance*, 40(1), 346-363.

Diebold, F. X., & Yilmaz, K. (2009). Measuring financial asset return and volatility spillovers, with application to global equity markets. *Economic Journal*, 119(534), 158–171.

Diebold, F. X., & Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting*, 28(1), 57–66.

Dwyer, A., Gardner, G., and Willams, T., (2011). Global Commodity Markets - Price Volatility and Financialisation, *RBA Bulletin - Reserve Bank of Australia*, pp. 49-58.

Erb, C. B., & Harvey, C. R. (2006). The tactical and strategic value of commodity futures. *Financial Analysts Journal*, 62(2), 69–97.

Erb, C. B., & Harvey, C. R. (2016). Conquering misperceptions about commodity futures investing. *Financial Analysts Journal*, 72(4), 26–35.

Gabauer, D., (2021). Dynamic measures of asymmetric & pairwise connectedness within an optimal currency area: Evidence from the ERM I system. *Journal of Multinational Financial Management*, 60.

Gagnon, M-H., Manseau, G., & Power, G.J., (2020). They're back! Post-financialization diversification benefits of commodities. International Review of Financial Analysis, 71.

Gardebroek, C., Hernandez, M.A., and Robles, M., (2016). Market interdependence and volatility transmission among major crops. *Agricultural Economics*, 47, 141-155.

Gorton, G., & Rouwenhorst, K. G. (2006). Facts and fantasies about commodity futures. Financial Analysts Journal, 62(2), 47–68.

Grubel, H., (1968). Internationally diversified portfolios: Welfare gains and capital flows. *American Economic Review*, 58, 1299-1314.

Hachicha, N., Ben Amar, A., Ben Slimane, I., Bellalah, M., & Prigent, J-L., (2022). Dynamic connectedness and optimal hedging strategy among commodities and financial indices. International Review of Financial Analysis.

Hammoudeh, S., Chen, L., & Fattouh, B. (2010). Asymmetric adjustments in oil and metals markets. *The Energy Journal*, 31(4), 183-203.

He, Y., and Lin, B., (2018). Time-varying effects of cyclical fluctuations in China's energy industry on the macro economy and carbon emissions, *Energy*, 155, 1102-1112.

Isleimeyyeh, M., (2020), The role of financial investors in determining the commodity futures risk premium, *Journal of Futures Markets*, 40, 1375-1397.

Jacomy, M., Venturini, T., Heymann, S., and Bastian, M., (2014), ForceAtlas2, a Continuous Graph Layout Algorithm for Handy Network Visualization Designed for the Gephi Software. *Plos One*.

Kellner, R. & Rösch, D., (2019). A country specific point of view on international diversification. *Journal of International Money and Finance*, 98.

Koop, G., & Korobilis, D., (2014). A New Index of Financial Conditions. *European Economic Review*, 71, 101–116.

Koop, G., and Korobilis, D., (2010). Bayesian and multivariate time series methods for empirical macroeconomics, *Foundations and Trends in Econometrics*, 3, 267-358.

Koop, G., Leon-Gonzalez, R., and Strachan, R., (2009). On the evolution of the monetary policy transmission mechanism, *Journal of Economic Dynamics and Control*, 33(4), 997–1017.

Koop, G., Pesaran, M.H., and Potter, S.M., (1996), Impulse Response Analysis in Non-linear Multivariate Models, *Journal of Econometrics*, 74(1), 119-147.

Levy, H. and Sarnat, M., (1970). International diversification of investment portfolios. *American Economic Review*, 60, 668-675.

Markowitz, H., (1952). Portfolio Selection. *The Journal of Finance*, 7(1), 77-91.

Markowitz. H., (1959). *Portfolio Selection: Efficient Diversification of Investments*. Yale University Press. 368 pages.

Nakajima, J., (2011). Time-varying parameter VAR model with stochastic volatility: An overview of methodology and empirical aplications, *Monetary And Economic Studies*, 107-142.

Pesaran, H.H., and Shin, Y., (1998). Generalized Impulse response Analysis in Linear Multivariate Models, *Economics Letters*, 58(1), 17-20.

Phylaktis, K. and Ravazzolo, F., (2005). Stock market linkages in emerging markets: implications for international portfolio diversification. *Journal of International Financial Markets, Institutions and Money*, 15, 91-106.

Primiceri, G.E., (2005). Time-varying structural vector autoregressions and monetary policy, *Review of Economic Studies*, 72(3), 821-852.

Sims, C.A., (1980). Macroeconomics and reality, *Econometrica*, 48(1), 1-48.

Sims, C.A., (2001). Comment on Sargent and Cogley's 'Evolving Post-World War II U.S. Inflation Dynamics', *NBER Macroeconomics Annual*, 373-379.

Solnik, B.H., (1974). An equilibrium model of the international capital market. *Journal of Economic Theory*, 8(4), 500-524.

Wold, H. (1954). *A study in the analysis of stationary time series*. Almqvist & Wiksell.

Contact : www.lefmi.fr